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Mathematics 13

UNITS 1 - 6

Distance Learning

Alberta EDUCATION

Learning Facilitator's Manual

Mathematics 13

LEARNING FACILITATOR'S MANUAL

Note

This Mathematics 13 Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final tests until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

Mathematics 13
Learning Facilitator's Manual
Units 1-6
Alberta Distance Learning Centre
ISBN No. 0-7741-0585-2

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Teachers

Register with the Alberta Distance Learning Centre

The Alberta Distance Learning Centre is dedicated to upgrading and continually improving your learning facilitator's manual so that it accurately reflects any necessary revisions we have had to make in the student module booklets or the assignment booklets. The types of revisions that will be made are those that make the course more accurate, current, or more effective.

The ADLC will send you the latest enhancements and upgrades for your learning facilitator's manual if you return the following registration card to: Alberta Distance Learning Centre, Box 4000, Barrhead, Alberta, T0G 2P0, Attention: Design Department.

ADLC Learning Facilitator's Manual
Registration Card

First Name _____ Surname _____

School Name _____ School Phone Number _____

School Address _____

City _____ Postal Code _____

Course Title _____ Approximate Date of Purchase _____

You can help ensure that distance learning courseware is of top quality by letting us know of areas that need to be adjusted. Call the Alberta Distance Learning Centre free of charge by using the RITE line and ask for the Editing Unit. Also, a teacher questionnaire has been included at the back of most learning facilitator's manuals. Please take a moment to fill this out.

We're looking forward to hearing from you!

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Teacher's Copy of the Final Test with Marking Guide

Final Test

Teacher Questionnaire

Introduction

A survey of these course materials will confirm that this new learning package has been specially designed for many kinds of teachers working in a variety of situations.

Which Category Do You Fit?

- ❒ Small Schools Teacher
  - ❒ inexperienced
  - ❒ experienced, but in other subject areas
  - ❒ experienced in teaching Mathematics 13 but wanting to try a different approach
- ❒ Distance Learning Teacher
  - ❒ travelling to schools within the jurisdiction
  - ❒ using facsimile and teleconferencing to teach students within the area
- ❒ Larger Schools Teacher
  - ❒ inexperienced
  - ❒ experienced in teaching Mathematics 13 but wanting to try a different approach

Because these materials have been created by experienced classroom teachers and distance learning specialists, they have many advantages for students and teachers regardless of their situations.

Advantages for Students

- incorporates a strong learner-centred philosophy
- promotes such qualities in the learner as autonomy, independence, and flexibility
- is developed through media which suit the needs and circumstances of the learner
- reflects the experiential background of Alberta students
- opens up opportunities by overcoming barriers that result from geographical location
- promotes individualized learning, allowing learners to work at their own pace

Advantages for Teachers

- allows teachers maximum teaching time and minimizes preparation time
- includes different routes through the materials to suit different learners
- incorporates a wide range of teaching strategies, in particular those using independent and individual learning
- delivers curriculum designed by education specialists that reflects the Alberta Education Program of Studies with an emphasis on Canadian content
- provides learning materials which are upwardly compatible with advanced educational technology

Does it sound like something you could use?

The student materials are not the only components designed for independent, guided instruction; so is this Learning Facilitator's Manual. It begins with an overview of the current Alberta Education Program of Studies for high school mathematics. This summary is included for inexperienced teachers or those teachers who have found themselves teaching high school mathematics when their training is in other subject areas. This brief summary is not meant to replace the Alberta Education Program of Studies, but rather to help teachers confirm the highlights of the program.

Other parts of this introduction have also been included to help teachers become familiar with this new courseware and determine how they might want to use it in their classroom.

Beyond the introduction the guide itself contains answers, models, explanations, and other tips generated by the teachers who authored this course.

The courseware and LFM are the products of experienced classroom teachers and distance learning specialists. It is the hope of these teachers that their experience can be shared with those who want to take advantage of it.

Overview of the Program of Studies

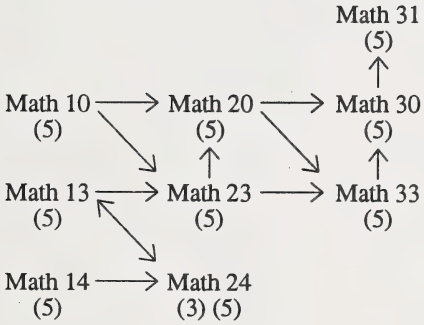
A knowledge of mathematics is essential in many other areas of learning. With available information and the use of technology, the emphasis will not be so much on memorizing formulas but on how to use mathematical information to solve problems. Students will be expected to construct, investigate, translate, discuss, analyse, and formulate mathematics in a dynamic way.

The Mathematics 10 – 20 – 30 – 31 sequence is designed for students with an interest and aptitude in mathematics who are intending to pursue postsecondary studies at a university or in a mathematics-intensive program at a technical school or college. Having successfully completed Mathematics 30, students will have fulfilled the mathematics requirement for the Advanced High School Diploma.

The Mathematics 13 – 23 – 33 sequence is the mainstream mathematics program designed for students who require mathematics to prepare for universities, colleges, trades, and employment. The Mathematics 13 – 23 – 33 sequence will satisfy the requirements for the General High School Diploma.

The Mathematics 14 – 24 program is a general series suitable for students who have experienced difficulties in previous mathematics courses.

The complete high school mathematics program is shown along with recommended transfer points. The number in brackets indicates credits.



This mathematics course is divided into units. Each unit covers a major area of mathematical study and is divided further into specific content areas called topics. Each unit has its own assignment booklet which the student completes and submits at the end of that unit.

Some of the distance learning courses are divided into sections within modules. If you receive letters or other materials from the Alberta Distance Learning Centre which refer to modules, please remember that a module is the same as a unit, and a section is the same as a topic.

Module = Unit

Section = Topic

Overview of Mathematics 13

Unit 1: Number Systems

Unit 2: Polynomials and Factoring

Unit 3: Geometry

Unit 4: Coordinate Geometry and Graphing

Unit 5: Relations

Unit 6: Statistics

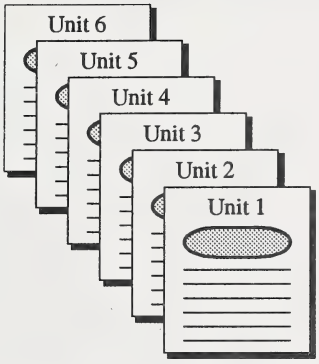
The Mathematics 13 course is the first course in the Mathematics 13 – 23 – 33 general series. The course is divided into six units as outlined. The course is offered for five credits only.

Structure of the Learning Package

Basic Design

This new learning package involves many other components in addition to the Learning Facilitator's Manual.

Units



The print components involve many booklets called units. These units contain guided activities that instruct students in a relevant, realistic setting.

The units have been specially designed to promote such qualities in the learner as autonomy, independence, and flexibility. Writers have incorporated such teaching strategies as working from the concrete to the abstract, linking the old to the new, getting students actively involved, and using advance, intermediate, and post organizers. Many other techniques enable learners to learn on their own for at least some of the time.

Contents at a Glance
Unit Overview
What You Already Know
Review
Topic 1 Activity 1 Activity 2 etc.
Topic 2 Activity 1 Activity 2 etc.
Topic 3 Activity 1 Activity 2 etc.
Unit Summary
Appendices

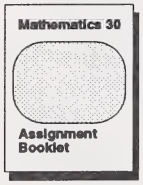
The structure of the unit booklets follows a systematic design. Each unit begins with a detailed table of contents which shows the students all the main steps. It acts as an organizer for students. The unit overview introduces the unit topic or theme. A graphic representation has been included to help visual learners and poor readers. The course overview states the weightings of each assignment.

The body of the unit is made up of two or more closely related topics. Each topic contains student activities that develop skills and knowledge centred around an objective.

The activities may involve print, audio, video, and computer formats. At times the student and the learning facilitator are allowed to choose the activity that best suits the student's needs and interests. Other activities such as the Extra Help and Extensions are optional pathways. This flexibility caters to each student's personal situation.

The summary focuses on the skills and strategies that the student has learned.

Assignment Booklet



Accompanying each unit is an assignment booklet. The activities in these booklets can be used for formative and for summative assessments. The students should complete these assignment booklets when they have thoroughly reviewed the unit materials. The assignment booklets have been designed for classroom use, for faxing, or for mailing. **If the booklets are not being mailed, you should remove the outside cover.**

Media

COMPUTER DISK

VIDEOCASSETTE

AUDIOCASSETTE

The package also includes reference to media. Pathways have been developed so students can use a variety of media to achieve the objective. These different routes have been included to suit different learners. Wherever videos or computer software have been included, a print pathway is also available. This way, if the media resource isn't available or desired, a student can follow the print pathway and still successfully achieve the objective.

Special audiocassettes guide the student through a difficult concept. The appearance of the audiocassette icon reminds students that there is this additional help available.

If the students are working individually, you may find this cassette a valuable asset. If you are working in a large group, you may wish to guide the students yourself.

Materials, Media, and Equipment

Mandatory Components

Equipment (Hardware)	Media	Materials
• audiocassette player	• prepared audiocassettes (come with learning package)	• LFM for Mathematics 13 • one complete set of unit booklets (6) and assignment booklets (6) for each student • geometry set • scientific calculator or graphics calculator • There is a final test.

Optional Components

Equipment (Hardware)	Media	Materials
• VCR	• videocassettes Videocassettes used in the course may be available from the Learning Resources Distributing Centre or ACCESS Network. You may also wish to call your regional library service for more information.	

Using This Learning Package in the Classroom

Conventional Classroom

Whether your classroom has desks in rows or tables in small groups, you may be most comfortable with a learning system that you can use with all your students in a paced style. In other words, you may want a package that will suit all of your students, so they can move through the materials as one group or several small groups. Because these materials contain different routes or pathways within each unit, they can address various learning styles and preferences. The materials also include many choices within the activities to cater to different thinking levels and ability levels. Because of their versatility and flexibility, these materials can easily suit a conventional classroom.

Open-Learning Classroom

Open learning is the concept of opening up opportunities by overcoming barriers of time, pace, and place by giving the learners a package specially designed to enable them to learn on their own for at least some of the time.

Such a concept is not new. Many teachers can recite attempts to establish an individualized learning system as they recognized the importance of trying to personalize courseware to meet each individual student's needs. But these efforts often failed due to lack of time and lack of quality materials that conformed to Alberta specifications.

Due to advanced educational technology and improved Alberta-specific learning packages, a student-centred approach is now possible. Improved technology now allows us to provide support to learners individually regardless of their pace or location. A teacher cannot be in twenty-eight places at one time offering guidance. However, media and a well-designed learning package can satisfy individual needs. Technology can also help provide an effective management system needed to track the students as they progress independently through the materials.

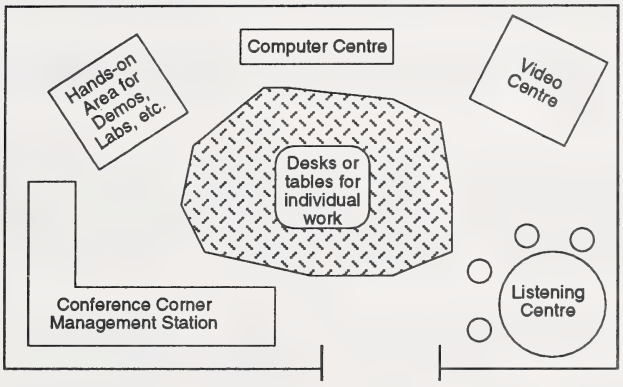
The key to a successful open-learning system depends on three vital elements: a learning package specially designed to enable students to learn effectively on their own for at least some of the time; various kinds of learner support; and a management system and style that ensures that the open-learning system runs smoothly.

The Key to a Successful Open-Learning System

Learning Package

The specially designed learning package needed for a successful open-learning system has been developed for you. The objectives teach current Alberta specifications using strategies designed for individualized instruction. As the learning facilitator, you need to be sure to have all the components in the learning package available to students as needed.

If adequate numbers of media are available to satisfy the demand, a centre can be established for specific media.



You may not have the luxury to have enough hardware to set up a permanent video or computer centre in your classroom. In that case, students should be encouraged to plan ahead. Perhaps every three to five days they should preview their materials and project when they would need a certain piece of media. This would allow you to group students, if necessary, or reserve media as required.

Support

Support is definitely a key element for successful learning, and when you're planning an individualized, non-paced program, you need to carefully plan when and how support will be given.

The materials contain a form of consistent support by providing immediate feedback for activities included in the unit booklet. High school students have solutions, models, explanations, and guides included in the appendix of every unit booklet. These are included so students can receive immediate feedback to clarify and reinforce their basic understanding before they move on to higher levels of thinking.

As the learning facilitator, you may be needed to offer more personal guidance to those students having difficulty, or you may need to reinforce the need for students to do these activities carefully before attempting the assignments in the assignment booklet.

The activities include choices and pathways. If a student is having difficulty, you may need to encourage that student to work on all the choices rather than one. This would provide additional instruction and practice in a variety of ways.

Another form of support is routine contact with each individual. This might be achieved with a biweekly conference scheduled by you, or as students reach a certain point (e.g., after each section is completed), they may be directed to come to the conference area.

Special counselling may be needed to help students through difficult stages. Praise and encouragement are important motivators, particularly for those students who are not used to working independently.

Direct teaching may be needed and scheduled at certain points in the program. This might involve small groups or a large group. It might be used to take advantage of something timely (e.g., election, eclipse, etc.), something prescheduled like the demonstration of a process, or something involving students in a hands-on, practical experience.

Support at a distance might include tutoring by phone, teleconferencing, faxing, or planned visits. These contacts are the lifeline between learners and distance education teachers, so a warm dialogue is essential.

Management

Good management of an open-learning system is essential to the success of the program. The following areas need action to ensure that the system runs smoothly:

- Scheduling, Distributing, and Managing Resources – As discussed earlier, this may require a need for centres or a system for students to project and reserve the necessary resources.

- Scheduling Students – Students and teachers should work together to establish goals, course completion timelines, and daily timelines. Although students may push to continue for long periods of time (e.g., all morning), teachers should discourage this. Concentration, retention, and motivation is improved by taking scheduled breaks.

- Monitoring Student Progress – You will need to record when units are completed by each student. Your data might also include the projected date of completion if you are using a student contract approach.

Sample of a Student Progress Chart

Mathematics 30		Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Final Test
Billy Adams	P							
	A							
Louise Despins	P							
	A							
Violet Klassian	P							
	A							
P = Projected Completion Date A = Actual Completion Date								

The student could keep a personal log as well. Such tracking of data could be stored easily on a computer.

- Recording Student Assessments – You will need to record the marks awarded to each student for work completed in each unit assignment booklet. The marks from these assignment booklets will contribute to a portion of the student's final mark. Other criteria may also be added (a special project, effort, attitude, etc.). Whatever the criteria, they should be made clear to all students at the beginning.

Sample of a Student Assessment Chart

Mathematics 30	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Year's Average	Final Test	Final Mark
Billy Adams	67	65	54	47	78	67	63		
Louise Despins	43	50	54	55	48	42	49		
Violet Klassian	65	65	66	66	67	69	67		

Letter grading could easily be substituted.

- Recording Effectiveness of System – Keep ongoing records of how the system is working. This will help you in future planning.

Sample of a System Assessment Chart

Unit 1			
Date	Unit Booklet	Assignment Booklet	Resources/Media

The Role of the Teacher in an Open-Learning Classroom

The teachers in a conventional classroom spend a lot of time talking to large groups of learners. The situation in open learning requires a different emphasis. Teachers will probably meet learners individually or in very small groups.

With this approach it is necessary to move beyond the idea of a passive learner depending largely on a continually supportive teacher. The teacher must aim to build the student's confidence, to stimulate the learner into self-reliance, and to guide the learner to take advantage of routes that are most meaningful and applicable to the learner.

These materials are student-centred, not teacher-centred. The teacher needs to facilitate learning by providing general support to the learner.

Evaluation

Evaluation is important to the development of every learner. Data gathering and processing, and decision making, at the student and teacher level, serve as means of identifying strengths and weaknesses.

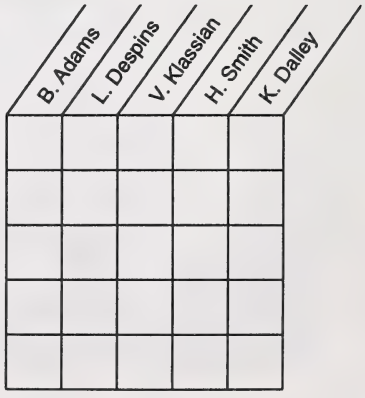
These specially designed learning packages contain many kinds of informal and formal evaluation.

Observation

In the classroom the teacher has the opportunity to see each student perform every day and to become aware of the level and nature of each student's performance.

Observations are more useful if they are recorded in an organized system. The following list of questions is a sample of types of observations and how they can be collected.

Observation Checklist



1. Does the student approach the work in a positive manner?
2. Is the student struggling with the reading level?
3. Does the student make good use of time?
4. Does the student apply an appropriate study method?
5. Can the student use references effectively, etc.?

Observation may suggest a need for an individual interview with a student.

Individual Conferences

Individual conferences may be paced (scheduled) by the calendar or at certain points in the unit, or they may be set up only as needed or requested.

During these conferences teachers can determine the student's progress and can assess the student's attitudes toward the subject, the program, school, and self, as well as the student's relationship with other students. With guided questions the teacher can encourage oral self-assessment; the student can discuss personal strengths or weaknesses in regard to the particular topic, unit, or subject area.

Self-Appraisal

Self-appraisal helps students recognize their own strengths and weaknesses. Through activities that require self-assessment, students also gain immediate feedback and clarification at early stages in the learning process. Teachers need to promote a responsible attitude toward these self-assessment activities. Becoming effective self-assessors is a crucial part of becoming an autonomous learner. By instructing, motivating, providing positive reinforcement, and systematically supervising, the learning facilitator will help students develop a positive attitude toward their own progress.

For variation, students may be paired and peer-assessing may become part of the system. The teacher may decide to have the student self-assess some of the activities, have a peer assess other activities, and become directly involved in assessing the remainder of the activities.

When the activities have been assessed, the student should be directed to make corrections. This should be made clear to students right from the start. It is important to note the correct association between the question and the response to clarify understanding, aid retention, and be of use for study purposes.

Many of the activities include choices for the student. If the student is having difficulty, more practice may be warranted, and the student may need to be encouraged to do more of the choices.

Each topic within a unit includes additional types of activities called Extra Help and Extensions. Students are expected to be involved in the decision as to which pathway best suits their needs. They may decide to do both.

Self-appraisal techniques can also be introduced at the individual conferences. Such questions as the following might be included:

- What steps are you taking to improve your understanding of this topic?
- What method of study do you use most?
- How do you organize your material to remember it?
- What steps do you follow when doing an assignment in your assignment booklet?
- What could you do to become an even better reader?
- Do you have trouble following directions?
- Did you enjoy this unit?

A chart or checklist could be used for recording responses.

Informal Evaluation: Assignments

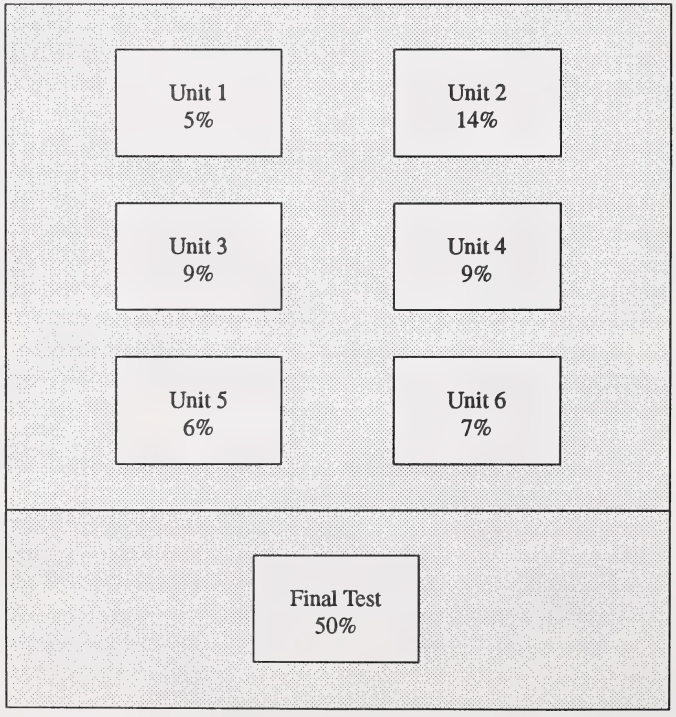
Informal evaluation, such as the assignments included in each unit, are an invaluable aid to the teacher. They offer ongoing assessment information about the student's achievement and the behaviour and attitudes that affect that achievement.

Each unit contains a separate booklet called the Assignment Booklet. This booklet assesses the knowledge or skills that the student has gained from the unit. **The student's mark for the unit may be based solely on the outcome of learning evident in the assignment booklet; however, you may decide to establish a value for other variables such as attitude or effort.** It is important that you establish at the beginning which outcomes will be evaluated, and that all students clearly understand what is expected.

Final Test

All LFMs include a formal final test which can be photocopied for each member of the class. The test, closely linked to the learning outcomes stated in the unit booklets, give the teacher precise information concerning what each student can or cannot do. Answers and explanations and marking guides are also included.

The value of the final test and each unit is the decision of the classroom teacher. Following is a suggestion only.



Introducing Students to the System

Your initiation to these learning materials began with a basic survey of what was included and how the components varied. This same process should be used with the class. After the materials have been explored, a discussion might include the advantages and the disadvantages of learning independently or in small groups. The roles of the students and teacher should be analysed. The necessary progress checks and rules need to be addressed. Your introduction should motivate students and build a responsible attitude toward learning autonomously.

Skill Level

It is important for students to understand that there are certain skills that they will need in order to deal successfully with the course materials. They are listed below:

- understanding and using instructional materials (table of contents, index, list of illustrations, and appendices)
- interpreting graphs and charts
- using reference materials
- recognizing special symbols
- using a scientific calculator

Other general skills are using reliable study methods, outlining, and learning to read at a flexible rate.

To decide the level and amount of instruction needed to accommodate the varied levels among students, you may wish to prepare and administer skill inventories or pretests. If most students need help with a particular skill, you may want to plan a total class instructional session. If only certain students lack a skill, you may want to set up a temporary skill group to help students who need it, or you may want to develop a skills file for this purpose.

Reading Level

These course materials are largely print based, but poorer readers need not be discouraged. It is important that you assure the students that these materials have been designed for easy reading. The authors have employed special strategies that lower and control the reading level. Some of them are

- the conscious selection of vocabulary and careful structuring of sentences to keep the materials at an independent reading level
- the integration of activities, examples, and illustrations to break text into appropriate-sized chunks
- the inclusion of many kinds of organizers (advance, graphic, intermediate, concept mapping, post organizers) to help give students a structure for incorporating new concepts

- the recognition that vocabulary and concepts are basic to understanding content materials and, thus, must be handled systematically (defined in context, marginal notes, footnotes, and often in a specialized glossary)

- the acknowledgement that background knowledge and experience play a vital role in comprehension

- the systematic inclusion of illustrations and optional videos to help poorer readers and visual learners, and audiocassettes and software as an alternative to print-based learning

- a variety of formats (paragraphs, lists, charts, etc.) to help poorer readers who do not absorb or retain main ideas easily in paragraph format

- the inclusion of media pathways and activity choices to encourage an active rather than passive approach

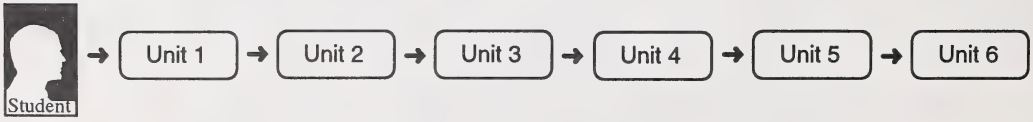
- instruction in a meaningful setting rather than in a contrived, workbook style

- using purposeful reading, viewing, and doing to produce better interpretation of the course materials

- the recognition that students need structured experiences when reading, viewing, or listening to instructional materials: developing pupil readiness, determining the purpose, providing guided instruction and feedback, rereading if necessary, and extending (This structure closely resembles the reading process.)

To help make the learning package more readable, you can begin in your unit preparation by reading (viewing, listening to) all the related materials that are going to be used. You need a solid background in order to assess and develop a background knowledge for students. The students' experiential bases may be assessed through brainstorming sessions concerning the topic or by using visuals and guided questions to predict what the topic might be about.

It is recommended that you start with Unit 1 because this unit includes basic introductory information, and it is also recommended that you end with Unit 6.



Mathematics 13

UNIT 1

Number Systems

Distance Learning

Alberta EDUCATION

Learning Facilitator's Manual

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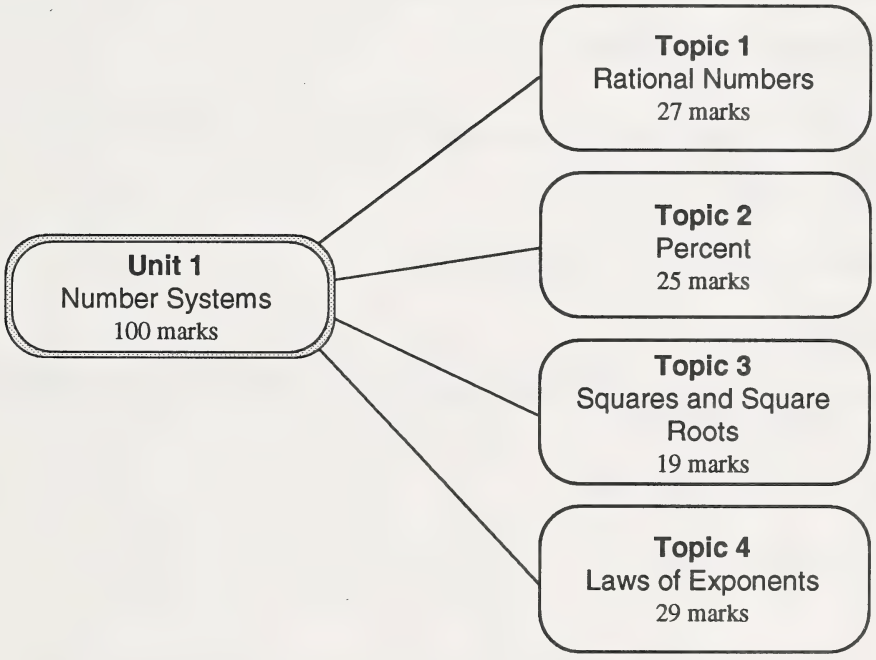
Unit 1 Overview

This unit called Number Systems is worth 10% of the final course mark.

The unit consists of four topics. Each topic has an assignment found in the Assignment Booklet.

Unit 1 Number Systems



Concept Introduction Possibilities

- Distinguish between rational and irrational numbers.
- Relate percents to decimals and fractions.
- Show why perfect squares are called squares, and what is meant by exact square roots and approximate square roots.
- Show why the Product Law and Quotient Law are what they are.

Possible Presentation Activities

- use of a simple Venn diagram
- use of a number line, positive tiles, and negative tiles
- use of calculators, tables, charts, etc.
- use of squares and rulers

These are suggestions only. Please use others if needed.

Marking Guidelines

Students have been requested to show their work for partial marks. Please use the marks assigned to each question as a **total** question mark.

An example of partial marking follows.

(3) Question: $1\frac{1}{3}-\frac{2}{3}$

Student response: $1\frac{1}{3}-\frac{2}{3}=\frac{4}{3}-\frac{2}{3}$

$=\frac{2}{9}$

Since the question is worth 3 marks, allow 1 or $1\frac{1}{2}$ marks for stating $\frac{4}{3}-\frac{2}{3}$. Because $\frac{2}{9}$ is incorrect, the student would receive 1 or $1\frac{1}{2}$ marks for this question.

Topic 1: Rational Numbers

(3) 1. Which of the following represents an integer?

a. $27 + 2$

b. $3\frac{1}{2}$

(c.) $92 - 30$

d. 0.561

The answer is c because $92 - 30 = 62$ and $I = \ldots -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3, \ldots$.

(3) 2. $1\frac{1}{3} - \frac{2}{3} = \frac{4}{3} - \frac{2}{3}$

$= \frac{4-2}{3}$

$= \frac{2}{3}$

(3) 3. $\frac{3}{5} \times \frac{3}{4} = \frac{3 \times 3}{5 \times 4}$

$= \frac{9}{20}$

(3) 4. Winston Lee was charged a total of \$29.50 for an oil change. The oil cost \$8.60 and the rest of the charge was for labour. How much was the labour charge?

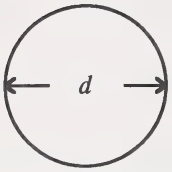
Total cost $= \$29.50$

Cost of oil $= -\$8.60$

Labour $= \$20.90$

The labour change was \$20.90.

(3) 5. The circumference of a circle is known to be 6.908 cm. Using the formula $C = \pi d$, calculate the diameter of the circle.



$C = \pi d$

$d = \frac{C}{\pi}$

$= \frac{6.908}{3.14}$

$= 2.2$

The diameter of the circle is 2.2 cm.

(2) 6. Karen Benson is planning a trip to Australia. When Karen looked up the exchange rate, she found the Australian dollar quoted at $1.0584 in Canadian dollars. Round this quote to the nearest cent.

greater than 5

$1.0584 = $1.06

(3) 7. Express $\frac{3}{5}$ as a decimal number.

$$\begin{array}{r} 0.6 \\ 5\overline{)3.0} \\ \underline{30} \\ 0 \end{array}$$

(3) 8. Express 0.125 as a fraction in simplest form.

$$0.125 = \frac{125}{1000}$$
$$= \frac{1}{8}$$

(4) 9. Simplify the expression $\frac{2}{3} \div 2 + \frac{1}{3} \times 4 + \frac{1}{3}$.

$$\frac{2}{3} \div 2 + \frac{1}{3} \times 4 + \frac{1}{3} = \frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{4}{1} + \frac{1}{3}$$
$$= \frac{1}{3} + \frac{4}{3} + \frac{1}{3}$$
$$= \frac{6}{3}$$
$$= 2$$

Topic 1

__27__ marks

Topic 2: Percent

(3) 1. Express $\frac{3}{4}$ as a percent.

$$\frac{3}{4}=\frac{3}{4}\times 100 \quad \text{or} \quad \frac{3}{4}=\frac{n}{100}$$
$$=\frac{300}{4} \qquad 4n=3\times 100$$
$$=75\% \qquad \frac{4n}{4}=\frac{300}{4}$$
$$n=75\%$$

(1) 2. a. Change 65% to a decimal number.

$$65\%=\frac{65}{100}$$
$$=0.65$$

(1) b. Change 4% to a decimal number.

$$4\%=\frac{4}{100}$$
$$=0.04$$

(2) 3. a. 1023 is 55% of what number?

$$\frac{1023}{x}=\frac{55}{100}$$
$$55x=1023\times 100$$
$$\frac{55x}{55}=\frac{102\,300}{55}$$
$$x=1860$$

(2) b. 42 is what percent of 210?

$$\frac{x}{100}=\frac{42}{210}$$
$$210x=42\times 100$$
$$\frac{210x}{210}=\frac{4200}{210}$$
$$x=20\%$$

(3) 4. In a mathematics class there are 24 students. If 16 of the students are girls, what percent of the class are boys?

Number of boys in the class is $24-16=8$.

$$\frac{8}{24}=\frac{x}{100}$$
$$24x=8\times100$$
$$\frac{24x}{24}=\frac{800}{24}$$
$$x=33\frac{1}{3}$$

In the class, $33\frac{1}{3}\%$ are boys.

(4) 5. A baseball team won 60% of their games. If they played 20 games in all, how many games did they win?

$$\frac{60}{100}=\frac{x}{20}$$
$$100x=60\times20$$
$$\frac{100x}{100}=\frac{1200}{100}$$
$$x=12$$

The baseball team won 12 games.

(2) 6. a. Express 1.05 as a percent.

$$1.05=1.05\times100$$
$$=105\%$$

(2) b. Express 0.41 as a percent.

$$0.41=0.41\times100$$
$$=41\%$$

(2) 7. 40% of 80 is what number?

$$\frac{40}{100} = \frac{x}{80}$$
$$100x = 40 \times 80$$
$$\frac{100x}{100} = \frac{3200}{100}$$
$$x = 32$$

(3) 8. Express $\frac{7}{16}$ as a percent.

$$\frac{7}{16} = \frac{x}{100}$$
$$16x = 7 \times 100$$
$$\frac{16x}{16} = \frac{700}{16}$$
$$x = 43.75\% \text{ or } 43\frac{3}{4}\%$$

$$\begin{array}{r} 43.75 \\ 16\overline{)700.00} \\ \underline{64} \\ 60 \\ \underline{48} \\ 120 \\ \underline{112} \\ 80 \\ \underline{80} \\ 0 \end{array}$$

Topic 2

__25__ marks

Topic 3: Squares and Square Roots

(2) 1. Evaluate 25^2.

$$25^2 = 25 \times 25$$
$$= 625$$

(4) 2. A square sheet of paper has a perimeter of 72 cm. Using the formula $P = 4s$ and the formula $A = s^2$, find the area of the sheet of paper.

Find the side length first using the perimeter formula.

$$P = 4s$$
$$72 = 4s$$
$$\frac{72}{4} = \frac{4s}{4}$$
$$s = 18 \text{ cm}$$

Next find the area using the area formula.

$$A = s^2$$
$$= 18 \text{ cm} \times 18 \text{ cm}$$
$$= 324 \text{ cm}^2$$

(2) 3. Evaluate $\sqrt{629}$ to the nearest whole number.

$$\sqrt{629} \doteq 25.079\,872\,41$$
$$\doteq 25$$

(3) 4. The area of a circle is 706.5 cm^2. Using the formula $A = \pi r^2$, find the radius of the circle where 3.14 is the value for π.

$$A = \pi r^2$$
$$706.5 = 3.14 r^2$$
$$\frac{706.5}{3.14} = \frac{3.14 r^2}{3.14}$$
$$r^2 = 225$$
$$\sqrt{r^2} = \sqrt{225}$$
$$r = 15$$

The radius of the circle is 15 cm.

(2) 5. Give the square roots of 81.

Since $81 = 9 \times 9$ and $81 = -9 \times -9$, the square roots of 81 are 9 and -9.

(1) 6. Determine the square of 15.

$15^2 = 15 \times 15$
$= 225$

(1) 7. 1, 4, 9, 16, and 25 are all examples of **perfect** squares.

(2) 8. Place whole numbers in the blanks to make a correct statement.

6 $< \sqrt{46} <$ **7**

Since $\sqrt{36} = 6$ and $\sqrt{49} = 7$ you get $\sqrt{36} < \sqrt{46} < \sqrt{49}$ and $6 < \sqrt{46} < 7$.

(2) 9. Evaluate $\sqrt{64 - 39}$.

$\sqrt{64 - 39} = \sqrt{25}$
$= 5$

Topic 3

19 marks

Topic 4: Laws of Exponents

(2) 1. Evaluate $(-41)^4$.

$$(-41)^4 = (-41)\times(-41)\times(-41)\times(-41)$$
$$= 2\,825\,761$$

(4) 2. A painting that sold for \$100.00 in 1900 was found to increase in value by 100% each decade. Its value in 1980 could be found by calculating $\$100.00\times 2^8$. Give the 1980 value of the painting.

$$\$100.00\times 2^8 = \$100.00\times 256$$
$$= \$25\,600.00$$

(2) 3. What is $17^{40}\times 17^2$ as a single power?

$$17^{40}\times 17^2 = 17^{40+2}$$
$$= 17^{42}$$

(3) 4. Evaluate $3^4\times 3^5$.

$$3^4\times 3^5 = 3^{4+5}$$
$$= 3^9$$
$$= 19\,683$$

(1) 5. Express each of the following as a single power:

a. $136^{28} \div 136^{14} = 136^{28-14}$

$= 136^{14}$

(1) b. $85^{10} \div 85^{15} = 85^{10-15}$

$= 85^{-5}$ or $\frac{1}{85^5}$

(2) 6. Evaluate $2^6 \div 2^3$.

$2^6 \div 2^3 = 2^{6-3}$

$= 2^3$

$= 8$

7. Evaluate the following:

(1) a. $35^0 =$ **1**

(2) b. $2^{-3} = \frac{1}{2^3}$

$= \frac{1}{8}$

(5) 8. Evaluate $\left(8^{-1} - 2^{-4}\right) \times 2 + 6 \div 3$.

$$\left(8^{-1} - 2^{-4}\right) \times 2 + 6 \div 3 = \left(\frac{1}{8} - \frac{1}{16}\right) \times 2 + 2$$

$$= \left(\frac{2}{16} - \frac{1}{16}\right) \times 2 + 2$$

$$= \left(\frac{2-1}{16}\right) \times 2 + 2$$

$$= \frac{1}{16} \times 2 + 2$$

$$= \frac{2}{16} + 2$$

$$= \frac{1}{8} + 2$$

$$= 2\frac{1}{8}$$

9. Evaluate the following.

(1) a. $(-3)^4 + 5^0 = (-3)\times(-3)\times(-3)\times(-3)+1$
$= 9\times 9+1$
$= 81+1$
$= 82$

(1) b. $(-3)^{-4} - 9^{-2} = \frac{1}{(-3)^4} - \frac{1}{9^2}$
$= \frac{1}{81} - \frac{1}{81}$
$= 0$

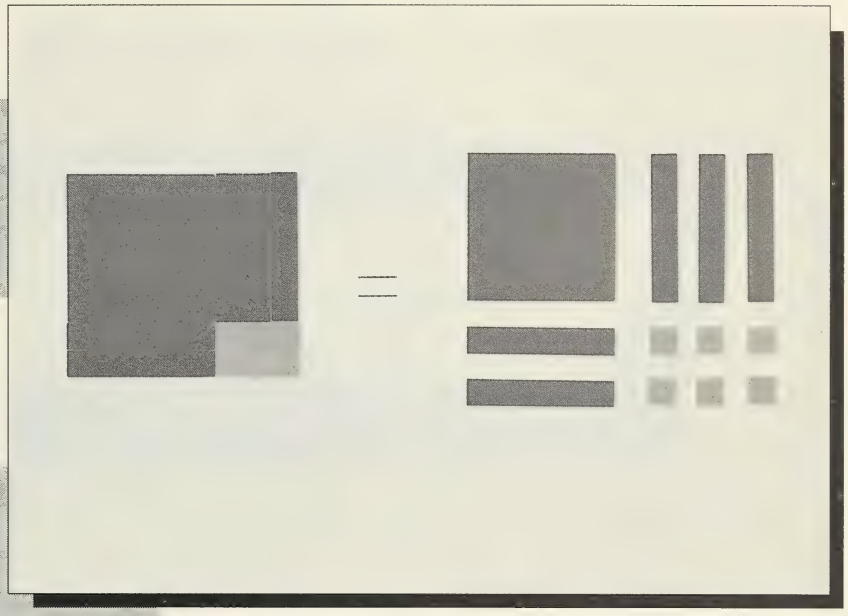
(2) 10. In 6^5, 6 is called the **base** and 5 is called the **exponent**.

(2) 11. Evaluate 0^0 = **undefined**

Topic 4

29 marks

Mathematics 13



UNIT 2

Polynomials & Factoring

Distance Learning

Alberta EDUCATION

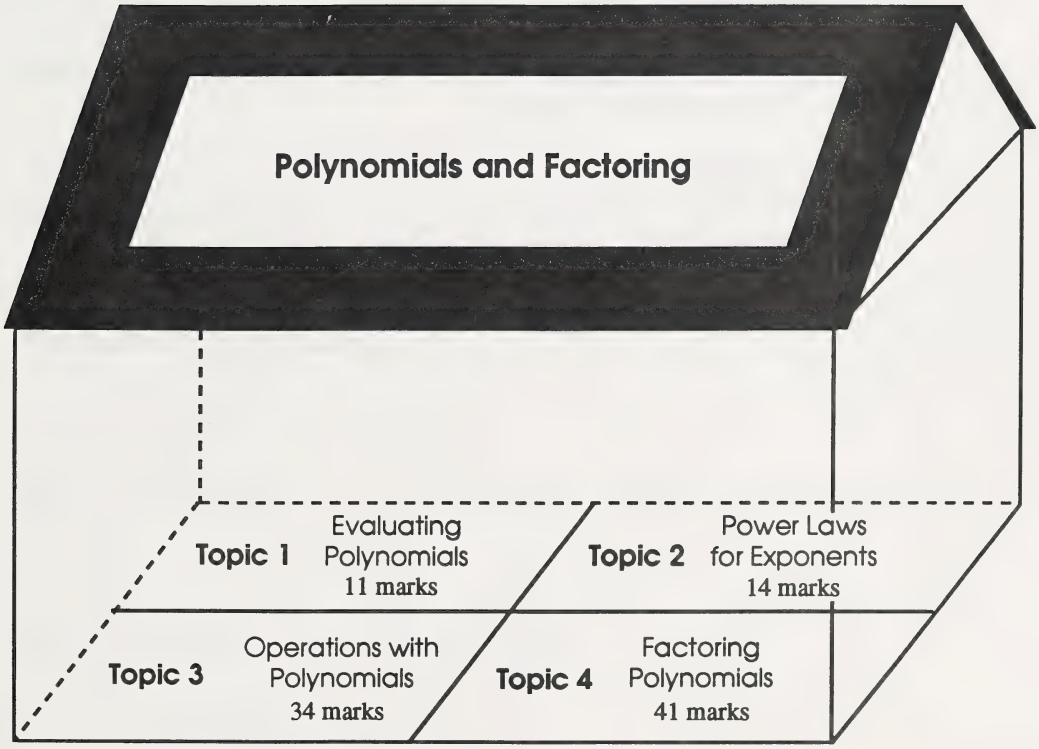
Learning Facilitator's Manual

Unit 2 Overview

This unit called Polynomials and Factoring is worth 28% of the course mark.

The unit consists of four topics. Each topic has an assignment found in the Assignment Booklet.

Unit 2



Concept Introduction Possibilities

- Substitute numerical values to evaluate polynomials.
- Use power laws to multiply and divide terms.
- Show how to combine like terms.
- Distinguish between expanding and factoring.
- Use polynomial tiles and binomial grids to find the expanded form of binomials.
- Compare algebraic factoring and factoring using a polynomial grid.

Possible Presentation Activities

- Using polynomial tiles and binomial grids, show how to find the area of a rectangle when given a length and a width.

- Using polynomial tiles and binomial grids, show how to find an unknown length of a rectangle when given the area and the width.

Marking Guidelines

- If the answer is correct, give full marks whether or not any work is shown.
- If the answer is incorrect, give partial marks for any correct work shown.

Topic 1: Evaluating Polynomials

1. Name the numerical coefficient in each term given.

(1) a. $-5x^2$

The numerical coefficient is -5.

(1) b. $2x^3y$

The numerical coefficient is 2.

(1) c. $-\frac{7}{2}y^4$

The numerical coefficient is $-\frac{7}{2}$.

(3) 2. Classify the polynomial $3a^2bc^3d$. Determine the special name, the degree, and the number of variables in the polynomial.

The special name of this polynomial is monomial.

The degree of this polynomial is 7.

$$a^2b^1c^3d^1 = 2+1+3+1$$
$$= 7$$

The number of variables in this polynomial is 4. The variables are a, b, c, and d.

3. Given $a=1$, $b=8$, and $c=4$, evaluate the following:

(1) a. $7ab^3$

$$7ab^3 = 7(1)\left(8^3\right)$$
$$= 7\times 512$$
$$= 3584$$

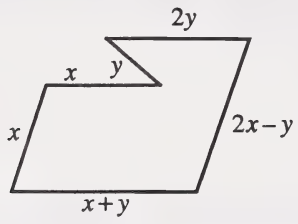
(1) b. $\sqrt{2bc}$

$$\sqrt{2bc} = \sqrt{2 \times 8 \times 4}$$
$$= \sqrt{64}$$
$$= 8$$

(1) c. $31a^2b + \frac{c}{b}$

$$31a^2b + \frac{c}{b} = 31(1)^2(8) + \frac{4}{8}$$
$$= 248 + 0.5$$
$$= 248.5 \text{ or } 248\frac{1}{2}$$

(2) 4. Write the expression for the perimeter of the following figure.



$$P = (x+y) + x + x + y + 2y + (2x - y)$$
$$= x + y + x + x + y + 2y + 2x - y$$
$$= x + x + x + 2x + y + y + 2y - y$$
$$= (5x + 3y) \text{ units}$$

Topic 1

11 marks

Topic 2: Power Laws for Exponents

1. Simplify the following and write your answers with only positive exponents.

(1) a. $y^4 \times y^3$

$$y^4 \times y^3 = y^{4+3}$$
$$= y^7$$

(1) b. $x^7 x^{-2}$

$$x^7 x^{-2} = x^{7-2}$$
$$= x^5$$

(1) c. $\dfrac{4m^5}{2m^2}$

$$\frac{4m^5}{2m^2} = \frac{4}{2} \times m^{5-2}$$
$$= 2m^3$$

(1) d. $\left(b^5\right)^7$

$$\left(b^5\right)^7 = b^{5\times 7}$$
$$= b^{35}$$

(2) e. $\left(b^0 \times b^6\right) + b^2$

$$\left(b^0 \times b^6\right) + b^2 = b^6 + b^2$$
$$= b^{6-2}$$
$$= b^4$$

(2) f. $\left(a^2 b^2\right)^4$

$$\left(a^2 b^2\right)^4 = a^{2\times4} b^{2\times4}$$
$$= a^8 b^8$$

(3) g. $\left(\frac{-x}{y}\right)^{-3}$

$$\left(\frac{-x}{y}\right)^{-3} = \left(\frac{y}{-x}\right)^3$$
$$= \frac{y^3}{-x^3} \text{ or } \frac{-y^3}{x^3}$$

(3) h. $\left(x^{-1} y^3 z\right)^{-2}$

$$\left(x^{-1} y^3 z\right)^{-2} = x^{-1\times(-2)} y^{3\times(-2)} z^{1\times(-2)}$$
$$= x^2 y^{-6} z^{-2}$$
$$= \frac{x^2}{y^6 z^2}$$

Topic 2

__14__ marks

Topic 3: Operations with Polynomials

1. Add the following polynomials. Leave your answer in alphabetical order and descending order of powers.

(2) a. $(2a-5b+c)+(a+3b-7c)+(2a-b)$

$(2a-5b+c)+(a+3b-7c)+(2a-b)=5a-3b-6c$

(2) b. $(3x^2+2xy)+(y^2-x^2)+(2y^2-5xy+x^2)$

$(3x^2+2xy)+(y^2-x^2)+(2y^2-5xy+x^2)=3x^2-3xy+3y^2$

2. Subtract the following polynomials.

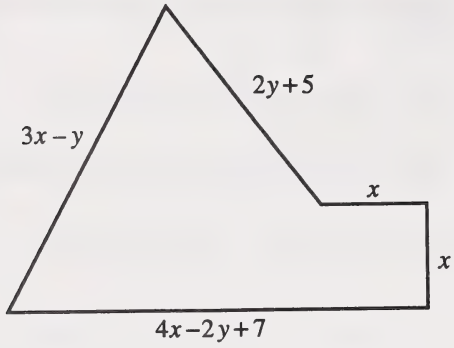
(2) a. $(6x-5y)-(3x+y)$

$$\begin{aligned}(6x-5y)-(3x+y) &= 6x-5y-3x-y \\ &= 6x-3x-5y-y \\ &= 3x-6y\end{aligned}$$

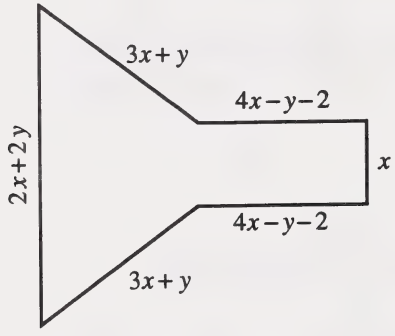
(2) b. $(5x^3-4x^2+1)-(x^3-4x^2)$

$$\begin{aligned}(5x^3-4x^2+1)-(x^3-4x^2) &= 5x^3-4x^2+1-x^3+4x^2 \\ &= 5x^3-x^3-4x^2+4x^2+1 \\ &= 4x^3+1\end{aligned}$$

⑥ 3. Find the difference in the perimeters of the following:



$$P_1 = (3x-y)+(2y+5)+x+x+(4x-2y+7)$$
$$= 3x+x+x+4x-y+2y-2y+5+7$$
$$= (9x-y+12) \text{ units}$$



$$P_2 = (3x+y)+(4x-y-2)+x+(4x-y-2)+(3x+y)+(2x+2y)$$
$$= 3x+4x+x+4x+3x+2x+y-y-y+y+2y-2-2$$
$$= (17x+2y-4) \text{ units}$$

Difference $= P_1 - P_2$
$= (9x-y+12)-(17x+2y-4)$
$= 9x-y+12-17x-2y+4$
$= (-8x-3y+16)$ units

or

$$\begin{array}{r} 9x - y + 12 \\ -\ 17x + 2y - 4 \\ \hline (-8x - 3y + 16) \text{ units} \end{array}$$

4. Expand each of the following:

(2) a. $-5k\left(k^2+4k-3\right)$

$$-5k\left(k^2+4k-3\right)=-5k^3-20k^2+15k$$

(4) b. $2x^{-1}\left(3x^2-4x-2\right)$

$$\begin{aligned}2x^{-1}\left(3x^2-4x-2\right)&=2x^{-1}\left(3x^2\right)+2x^{-1}(-4x)+2x^{-1}(-2)\\&=6x-8x^0-4x^{-1}\\&=6x-8-4x^{-1}\end{aligned}$$

5. Expand each of the following. Express each product in simplified form.

(1) a. $(a+b)(c+d)$

$$(a+b)(c+d)=ac+ad+bc+bd$$

(1) b. $(x+y)(x+y)$

$$\begin{aligned}(x+y)(x+y)&=x^2+xy+xy+y^2\\&=x^2+2xy+y^2\end{aligned}$$

(1) c. $(a-b)(a-b)$

$$\begin{aligned}(a-b)(a-b)&=a^2-ab-ab+b^2\\&=a^2-2ab+b^2\end{aligned}$$

(1) d. $(x+y)(x-y)$

$$\begin{aligned}(x+y)(x-y)&=x^2-xy+xy-y^2\\&=x^2-y^2\end{aligned}$$

(2) e. $(3k+4y)(2k+3y)$

$$(3k+4y)(2k+3y) = 6k^2 + 9ky + 8ky + 12y^2$$
$$= 6k^2 + 17ky + 12y^2$$

(2) f. $(5r+8t)(5r-8t)$

$$(5r+8t)(5r-8t) = 25r^2 - 40rt + 40rt - 64t^2$$
$$= 25r^2 - 64t^2$$

6. Divide each of the following.

(3) a. $\dfrac{6a^3b^2c + 12a^3c}{6a^3c}$

$$\frac{6a^3b^2c + 12a^3c}{6a^3c} = \frac{6a^3b^2c}{6a^3c} + \frac{12a^3c}{6a^3c}$$
$$= b^2 + 2$$

(3) b. $\dfrac{8x^3y^3 - 6x^2y^4 + 14xy^2}{2xy^2}$

$$\frac{8x^3y^3 - 6x^2y^4 + 14xy^2}{2xy^2} = \frac{8x^3y^3}{2xy^2} - \frac{6x^2y^4}{2xy^2} + \frac{14xy^2}{2xy^2}$$
$$= 4x^2y - 3xy^2 + 7$$

Topic 3

34 marks

Topic 4: Factoring Polynomials

1. Factor each of the following completely.

(1) a. $6x^2y-8xy^2$

$6x^2y-8xy^2=2xy(3x-4y)$

(1) b. $4-8y+12y^2$

$4-8y+12y^2=4\left(1-2y+3y^2\right)$

(1) c. $25x^4y^3+35x^2y^4$

$25x^4y^3+35x^2y^4=5x^2y^3\left(5x^2+7y\right)$

2. Factor each of the following completely using algebraic methods.

(1) a. $x^2+8x+12$

$x^2+8x+12=(x+6)(x+2)$

(2) b. $2x^2-4x-126$

$2x^2-4x-126=2\left(x^2-2x-63\right)$

$=2(x-9)(x+7)$

(4) c. $4(2x-y)^2-z^2$

$4(2x-y)^2-z^2=[2(2x-y)+z][2(2x-y)-z]$

$=(4x-2y+z)(4x-2y-z)$

Let $(2x-y)=d$ and substitute to get $4d^2-z^2$.
$4d^2-z^2=(2d+z)(2d-z)$
Substitute $(2x-y)$ for d to get $[2(2x-y)+z][2(2x-y)-z]$ and simplify to finish with $(4x-2y+z)(4x-2y-z)$.

(1) d. $x^2+14x+49$

$x^2+14x+49=(x+7)(x+7)$

(2) e. $4c^2-9d^2$

$4c^2-9d^2=(2c+3d)(2c-3d)$

(2) f. $16a^2b^2-9c^2$

$16a^2b^2-9c^2=(4ab+3c)(4ab-3c)$

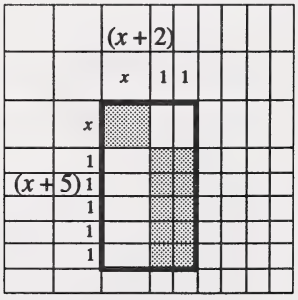
(4) g. x^4-1

$x^4-1=\left(x^2+1\right)\left(x^2-1\right)$

$=\left(x^2+1\right)(x+1)(x-1)$

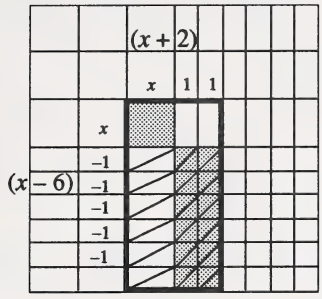
3. Factor each of the following polynomials using the binomial grid provided in each case.

(3) a. $x^2+7x+10$



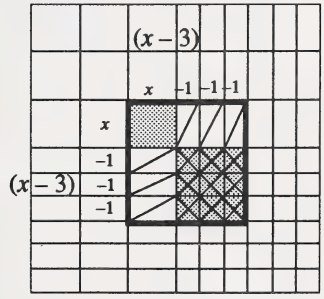
Therefore, $x^2+7x+10$ is $(x+2)(x+5)$ in factored form.

③ b. $x^2 - 4x - 12$



Therefore, $x^2 - 4x - 12$ is $(x+2)(x-6)$ in factored form.

③ c. $x^2 - 6x + 9$



Therefore, $x^2 - 6x + 9$ is $(x-3)(x-3)$ or $(x-3)^2$ in factored form.

(5) 4. A rectangular garden has an area of $(6x^3 - 9x^2 + 3x)$ square units. Find the length of the other side if one side has a length of $3x$ units. Draw and label a diagram to complete your solution.

$$A = l \times w$$

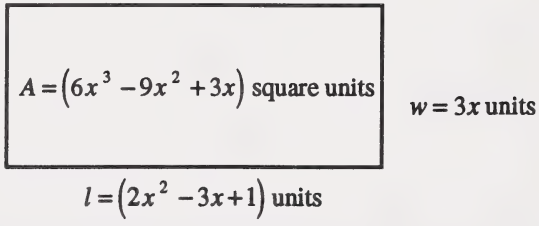
$$6x^3 - 9x^2 + 3x = (l)(3x)$$

$$\frac{(l)(3x)}{3x} = \frac{6x^3 - 9x^2 + 3x}{3x}$$

$$l = \frac{6x^3}{3x} + \frac{-9x^2}{3x} + \frac{3x}{3x}$$

$$l = 2x^2 - 3x + 1$$

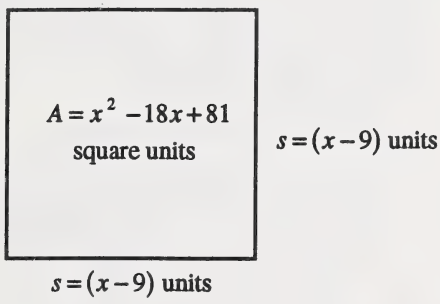
The length of the other side is $(2x^2 - 3x + 1)$ units.



(5) 5. A square swimming pool has an area of $(x^2 - 18x + 81)$ square units. Determine the length of each side of the swimming pool. (Hint: Factor $x^2 - 18x + 81$.) Draw and label a diagram to complete your solution.

$$x^2 - 18x + 81 = (x-9)(x-9)$$
$$= (x-9)^2$$

The pool is $(x-9)$ units wide and $(x-9)$ units long.



6. Fill the blanks in each statement.

(2) a. The polynomial $x^2 - 8x +$ **16** is a perfect square trinomial.

(1) b. The polynomial $x^2 - 16$ is an expression which shows the difference of **squares**.

Topic 4

41 marks

Mathematics 13

UNIT 3
Geometry

Distance Learning

Alberta EDUCATION

Learning Facilitator's Manual

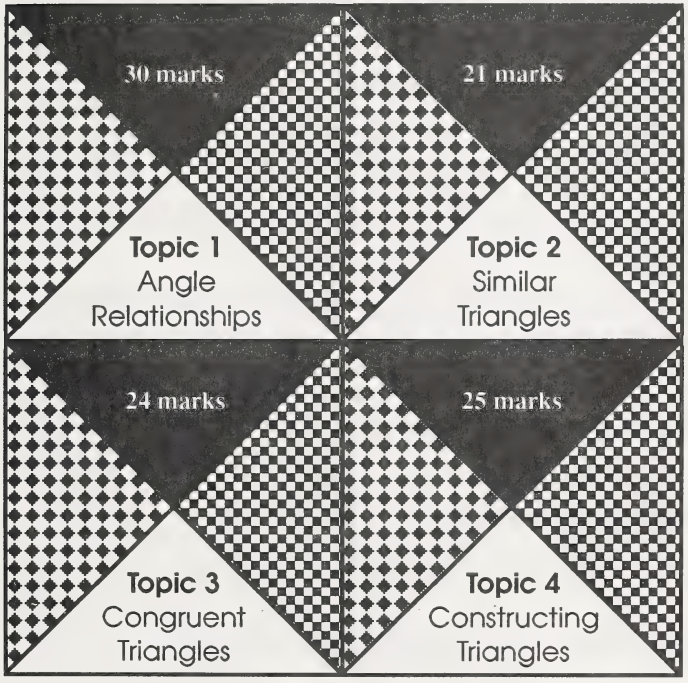
Cover photo: PHOTO SEARCH LTD.

Unit 3 Overview

This unit called Geometry is worth 18% of the course work.

The unit consists of four topics. Each topic has an assignment found in the Assignment Booklet.

Unit 3 Geometry



Concept Introduction Possibilities

- Show how angles formed by a transversal intersecting two parallel lines are related.
- Show that corresponding sides of two similar triangles are in proportion and may be used in indirect measurement.
- Show how to construct congruent triangles.
- Show how two triangles may be congruent when certain requirements are met.

Possible Presentation Activities

- Present concrete examples of parallel lines intersected by a transversal.
- Using an overhead projector, measure and compare the angles of similar triangles on the projector and on the screen.
- Have the students construct triangles of a given size and compare them by overlapping them on a classroom window.
- Using a picture of the house as it is found in the sport of curling, show how a compass may be used to determine which rock is closer to the button of the circle or the house.

Marking Guidelines

If the answer is correct, give full marks whether or not any work is shown.

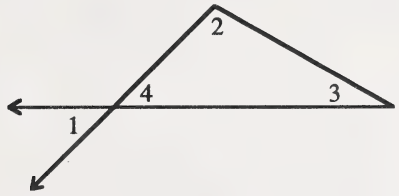
If the answer is incorrect, give partial marks for any correct work shown.

In constructions of triangles, arcs must be shown for marks to be given.

Topic 1: Angle Relationships

No protractors are allowed for any of the Topic 1 questions.

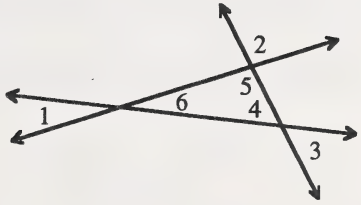
(3) 1. Use the given diagram. Suppose that $\angle 2 = 105°$ and $\angle 3 = 30°$. What is the measure of $\angle 1$? Explain your answer.



$$\begin{aligned}\angle 4 &= 180° - (\angle 2 + \angle 3) \\ &= 180° - (105° + 30°) \\ &= 45°\end{aligned}$$

Since vertically opposite angles are equal, $\angle 1 = \angle 4$.
Since $\angle 4 = 45°$ from the previous calculations, $\angle 1 = 45°$.

(3) 2. Use the given diagram. What is $\angle 1 + \angle 2 + \angle 3$? Explain your answer.



$\angle 6 + \angle 5 + \angle 4 = 180°$

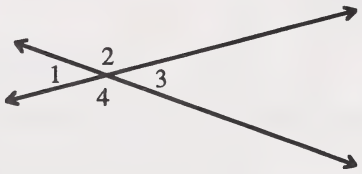
Each of the following pairs of angles are vertically opposite angles.

$\angle 4 = \angle 3$
$\angle 5 = \angle 2$
$\angle 6 = \angle 1$

Therefore, $\angle 1 + \angle 2 + \angle 3 = 180°$ by the process of substitution.

(3) 3. Use the following information and the diagram to find the measure of $\angle 3$. Explain your answer.

- $\angle 1 = 35°$
- $\angle 2 = 145°$
- $\angle 4 = 145°$



Since vertically opposite angles are equal, $\angle 3 = \angle 1$.
Therefore, since $\angle 1 = 35°$, it follows that $\angle 3 = 35°$.

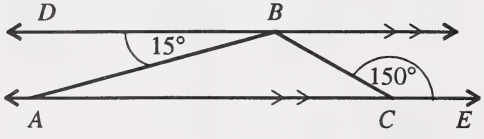
OR

Since $\angle 4$ and $\angle 3$ are supplementary angles, $\angle 3$ can be calculated as follows:

$$\angle 4 + \angle 3 = 180°$$
$$145° + \angle 3 = 180°$$
$$\angle 3 = 180° - 145°$$
$$= 35°$$
$$\therefore \angle 3 = 35°$$

Note that $\angle 2$ and $\angle 3$ can be used as the supplementary angles as well.

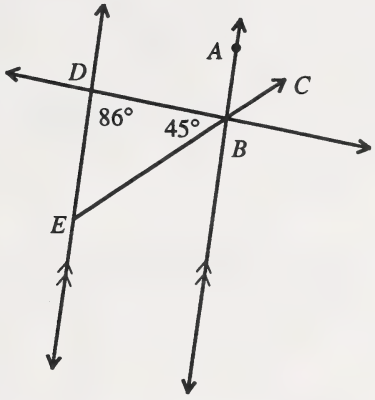
(3) 4. Use the following diagram. What is the measure of $\angle ABC$?



Since angles CBD and BCE are alternate angles and because $\overleftrightarrow{DB} \parallel \overleftrightarrow{AE}$, $\angle CBD = \angle BCE = 150°$.

$$\angle ABD + \angle ABC = \angle CBD$$
$$\angle ABC = \angle CBD - \angle ABD$$
$$= 150° - 15°$$
$$= 135°$$
$$\therefore \angle ABC = 135°$$

③ 5. Use the diagram. What is the measure of $\angle ABC$?



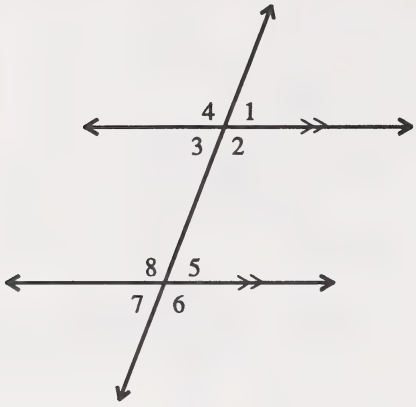
$$\begin{aligned}\angle BED &= 180° - (86° + 45°) \\ &= 180° - 131° \\ &= 49°\end{aligned}$$

$\overleftrightarrow{DE} \parallel \overleftrightarrow{AB}$

Since they are corresponding angles related to parallel lines, $\angle ABC = \angle BED = 49°$.

There are other ways to obtain the answer using supplementary angles and vertically opposite angles.

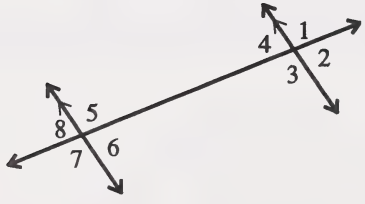
(3) 6. Use the diagram. Given that $\angle 2 = 111°$, what is the measure of $\angle 5$? Explain your answer.



The two horizontal lines are parallel as indicated by the symbol $\gg$ on the lines.
Since angles 5 and 2 are same-side interior angles, $\angle 5 + \angle 2 = 180°$.

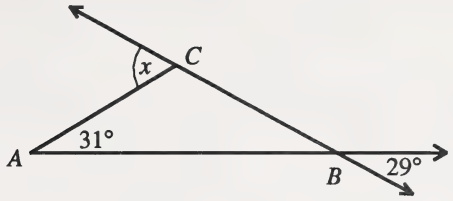
$$\begin{aligned}\angle 5 &= 180° - \angle 2 \\ &= 180° - 111° \\ &= 69°\end{aligned}$$

(2) 7. Use the diagram. Given that $\angle 5 = 102°$, what is the measure of $\angle 3$? Explain why.



Since there is a transversal intersecting two parallel lines and $\angle 3$ and $\angle 5$ are equal alternate angles, $\angle 3 = \angle 5 = 102°$.

④ 8. Use the diagram to complete the following statement. Explain your answer.



$x =$ **60** °

Since the angles are vertically opposite angles, $\angle ABC = 29°$.

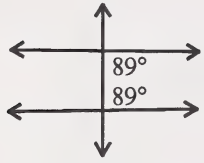
$$\begin{aligned}\angle ACB &= 180° - (31° + 29°) \\ &= 180° - 60° \\ &= 120°\end{aligned}$$

Since the angles are adjacent and form a straight angle, $x + \angle ACB = 180°$.

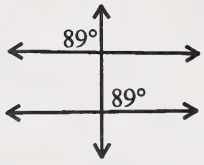
$$\begin{aligned}x &= 180° - \angle ACB \\ &= 180° - 120° \\ &= 60°\end{aligned}$$

(2) 9. Circle the letter of the correct answer. **In which diagram is there a pair of parallel lines represented?**

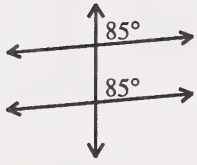
A.



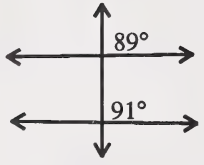
B.



(C.)

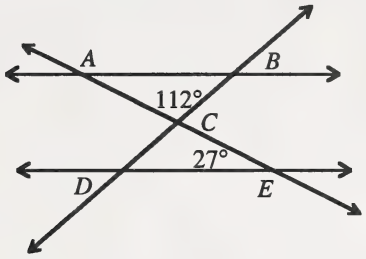


D.



The corresponding angles are equal in response C.

④ 10. Use the diagram. Line AB is parallel to line DE. Angle DEC measures 27 degrees and angle ACB measures 112 degrees. What is the measure of angle ABC? Explain your answer.



$\angle ABC =$ **41** degrees

Since angles BAC and DEC are alternate angles where $\overleftrightarrow{AB} \parallel \overleftrightarrow{DE}$, $\angle BAC = \angle DEC = 27°$.

Since the sum of the three angles in any triangle is 180°, $\angle ABC$ can be calculated as follows:

$$\begin{aligned}\angle ABC &= 180° - (\angle BAC + \angle ACB)\\ &= 180° - (27° + 112°)\\ &= 180° - 139°\\ &= 41°\end{aligned}$$

Therefore, the measure of $\angle ABC$ is 41°.

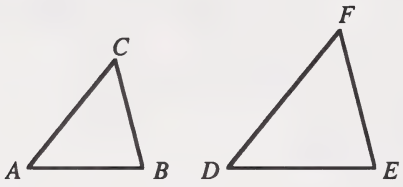
Topic 1

30 marks

Topic 2: Similar Triangles

(4) 1. Use the following information and the diagram to find the length of DF.

- $\Delta ABC \sim \Delta DEF$
- $AB = 800$ mm
- $DE = 1000$ mm
- $AC = 900$ mm

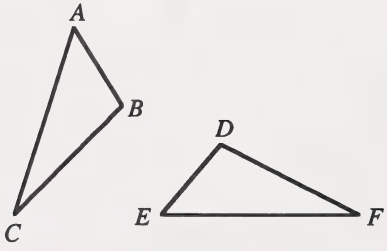


Let $DF = x$.

$$\frac{AB}{DE} = \frac{AC}{DF}$$
$$\frac{800}{1000} = \frac{900}{x}$$
$$800x = 900\,000$$
$$\frac{800x}{800} = \frac{900\,000}{800}$$
$$x = 1125$$

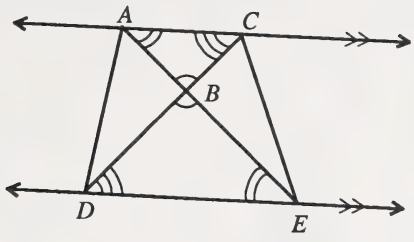
The length of DF is 1125 mm.

(2) 2. Use the diagram. Given that $\Delta ABC \sim \Delta EDF$, which angle corresponds to $\angle E$?



The angle that corresponds to $\angle E$ is $\angle A$.

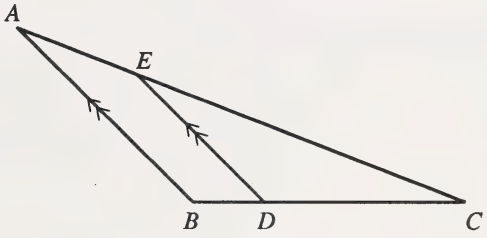
(4) 3. Use the diagram to complete the following.



$\Delta ABC \sim \Delta$ ___E___ ___B___ ___D___ (No other order of *EBD* is acceptable.)

Since $\overleftrightarrow{AC} \parallel \overleftrightarrow{DE}$, alternate interior angles have the same measure. Vertically opposite angles have the same measure too.

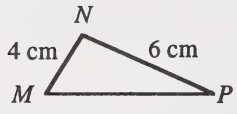
(2) 4. Use the diagram. Side *EC* corresponds to which side of ΔABC?

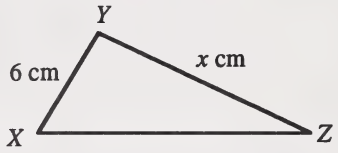


Because $\Delta ABC \sim \Delta EDC$, it follows that $\overline{AC}$ corresponds to $\overline{EC}$.

(3) 5. Use the following information and the diagram to find the length of *YZ*.

- $\Delta MNP \sim \Delta XYZ$
- $MN = 4$ cm
- $XY = 6$ cm
- $NP = 6$ cm





Let $YZ = x$.

$$\frac{MN}{XY} = \frac{NP}{YZ}$$

$$\frac{4}{6} = \frac{6}{x}$$

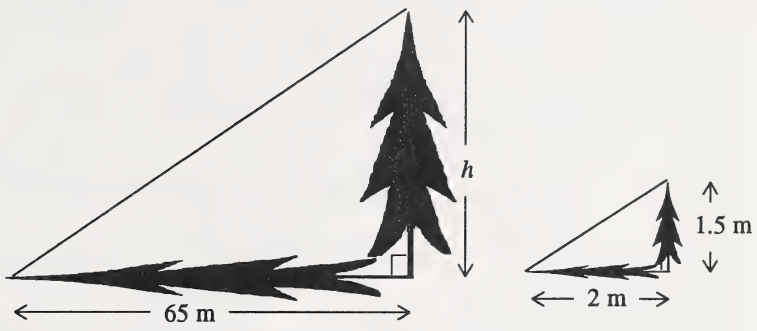
$$4x = 36$$

$$\frac{4x}{4} = \frac{36}{4}$$

$$x = 9$$

Therefore, $YZ = 9$ cm.

③ 6. Use the following diagram. At a certain time on a sunny day, two trees cast shadows of 2 m and 65 m. The smaller tree has a height of 1.5 m. The larger tree is a Douglas fir. What is the height of this Douglas fir? Round your answer to the nearest tenth of a metre.



Let h be the height of the taller tree.

$$\frac{h}{1.5} = \frac{65}{2}$$

$$2h = 97.5$$

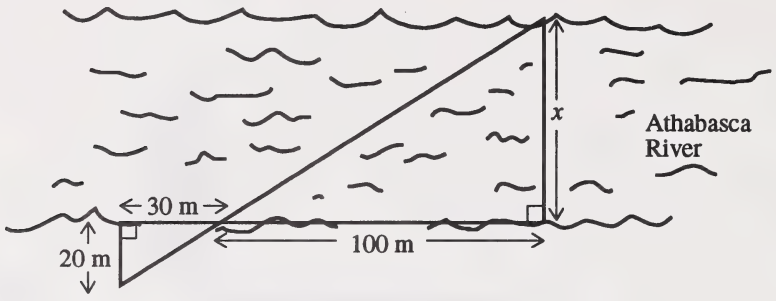
$$\frac{2h}{2} = \frac{97.5}{2}$$

$$h = 48.75$$

$$h \doteq 48.8$$

The height of the Douglas fir is approximately 48.8 m.

③ 7. Use the diagram that follows. Suppose the measurements on the diagram were made in order to indirectly determine the width of the Athabasca River at a certain spot. To the nearest metre, how wide is the river at that point?



Let x be the width of the river.

$$\frac{x}{20} = \frac{100}{30}$$

$$30x = 2000$$

$$x = 66.\overline{6}$$

$$\doteq 67 \text{ m}$$

Therefore, the Athabasca River is approximately 67 m wide at that particular spot.

Topic 2

__21__ marks

Topic 3: Congruent Triangles

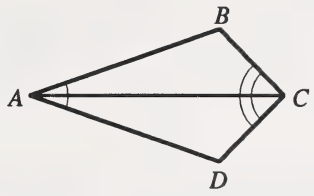
(2) 1. Use the diagram. Circle the letter of the correct answer.
By which congruence property is it true that $\Delta ABC \cong \Delta ADC$?

A. SSS

B. SAS

(C.) ASA

D. SSA



In ΔABC and ΔADC the following parts are congruent:

$\overline{AC} \cong \overline{AC}$	same side (S)
$\angle BAC \cong \angle DAC$	given (A)
$\angle BCA \cong \angle DCA$	given (A)

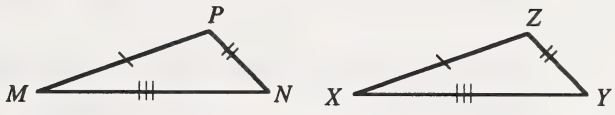
(2) 2. Use the diagram. Circle the letter of the correct answer.
By which congruence property is it true that $\Delta MNP \cong \Delta XYZ$?

(A.) SSS

B. SAS

C. ASA

D. none of the above



In ΔMNP and ΔXYZ the following parts are congruent:

$\overline{MP} \cong \overline{XZ}$	given (S)
$\overline{MN} \cong \overline{XY}$	given (S)
$\overline{NP} \cong \overline{YZ}$	given (S)

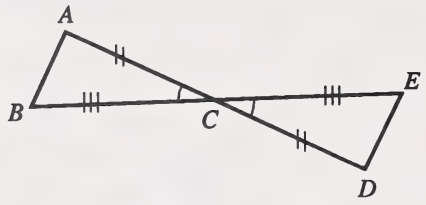
(2) 3. Use the diagram. Circle the letter of the correct answer.
By which congruence property is it true that $\Delta ABC \cong \Delta DEC$?

A. SSS

(B.) SAS

C. ASA

D. SSA



In ΔABC and ΔDEC the following parts are congruent:

$\overline{AC} \cong \overline{DC}$	given (S)
$\angle ACB \cong \angle DCE$	given (A)
$\overline{BC} \cong \overline{EC}$	given (S)

(3) 4. Two similar triangles that are the same size are **congruent** .
Explain your answer.

All corresponding angles are equal and all corresponding sides are equal. Since all properties satisfying congruency (SSS, SAS, or ASA) are met, the two triangles are congruent.

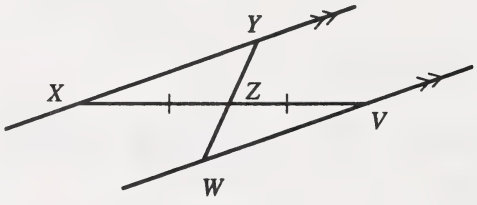
(2) 5. Circle the letter of the correct answer. By which congruence property is it true that $\Delta XYZ \cong \Delta VWZ$?

A. SSS

B. SAS

(C.) ASA

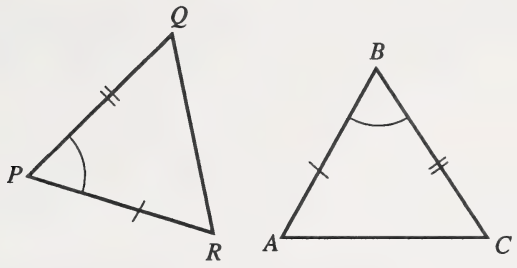
D. none of the above



In ΔXZY and ΔVZW the following parts are congruent:

$\overline{XZ} \cong \overline{VZ}$	given (S)
$\angle YZX \cong \angle WZV$	vertically opposite angles (A)
$\angle YXZ \cong \angle WVZ$	alternate angles for parallel line segments (A)

(4) 6. Use the diagram. Complete the following statement and show how you obtained your answer.



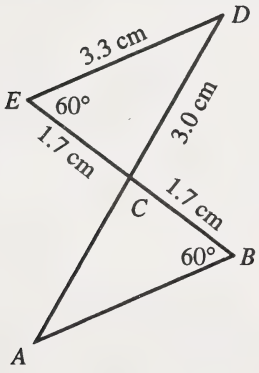
$\Delta ABC \cong$ $\underline{\Delta RPQ}$ (No other order for R, P, and Q is acceptable.)

In ΔABC and ΔRPQ the following parts are congruent:

$\overline{AB} \cong \overline{RP}$	given (S)
$\overline{BC} \cong \overline{PQ}$	given (S)
$\angle B \cong \angle P$	given (A)

This satisfies the SAS congruency property.

(4) 7. Use the diagram and the measures shown. What is the length of AB? Explain your answer.

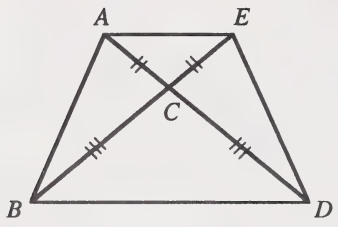


$AB = 3.3$ cm

Because the ASA congruency property is met, $\Delta DEC \cong \Delta ABC$.

$\overline{BC} \cong \overline{EC}$	same measure (S)
$\angle BCA \cong \angle ECD$	vertically opposite angles (A)
$\angle ABC \cong \angle DEC$	same measure (A)

(5) 8. Use the diagram that follows. Circle the letter of the correct answer.



Which of the following can be shown? Explain your answer.

A. $AE = AB$

(B.) $DE = BA$

C. $BE = AB$

D. not enough information in the diagram to show that any one of the above is true

Because the SAS congruency property is met, $\Delta ACB \cong \Delta ECD$.

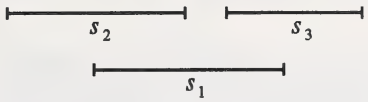
$\overline{AC} = \overline{EC}$	given (S)
$\angle ACB \cong \angle ECD$	vertically opposite angles (A)
$\overline{CB} \cong \overline{CD}$	given (S)

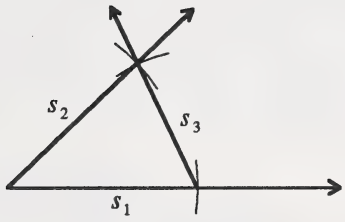
Topic 3

__24__ marks

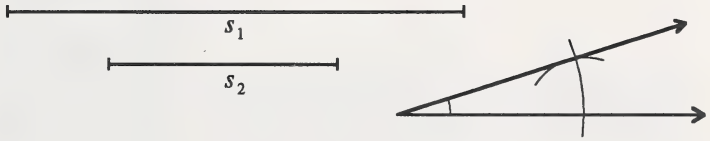
Topic 4: Constructing Triangles

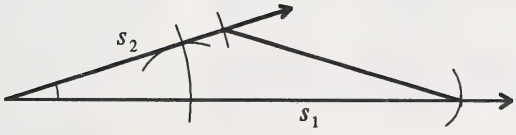
(6) 1. Using the SSS congruency property, construct a triangle using the line segments shown. Show all construction arcs clearly.



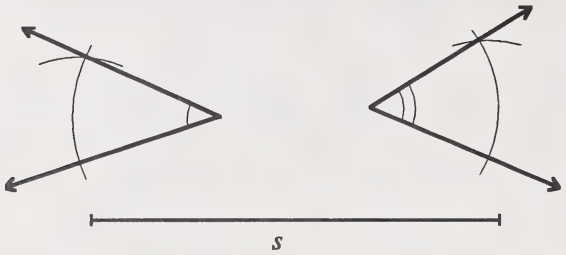


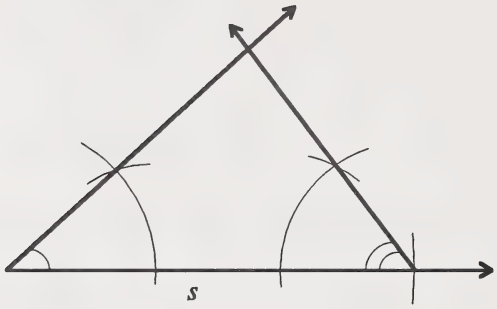
(6) 2. Using the SAS congruency property, construct a triangle using the elements shown. Show all construction arcs clearly.



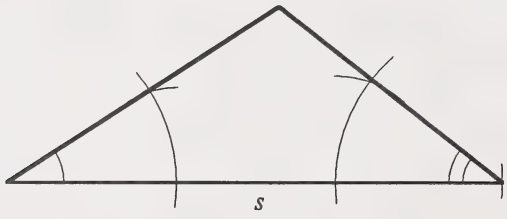


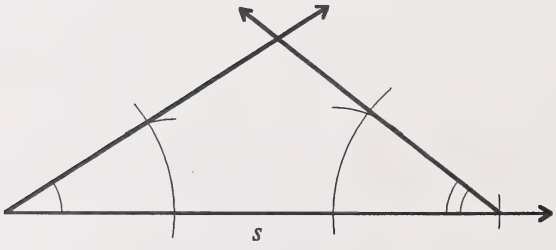
⑥ 3. Using the ASA congruency property, construct a triangle using the angles and the line segment given. Show all construction arcs clearly.





⑦ 4. Using the ASA congruency property, construct a triangle congruent to the given triangle. Show all construction arcs clearly.

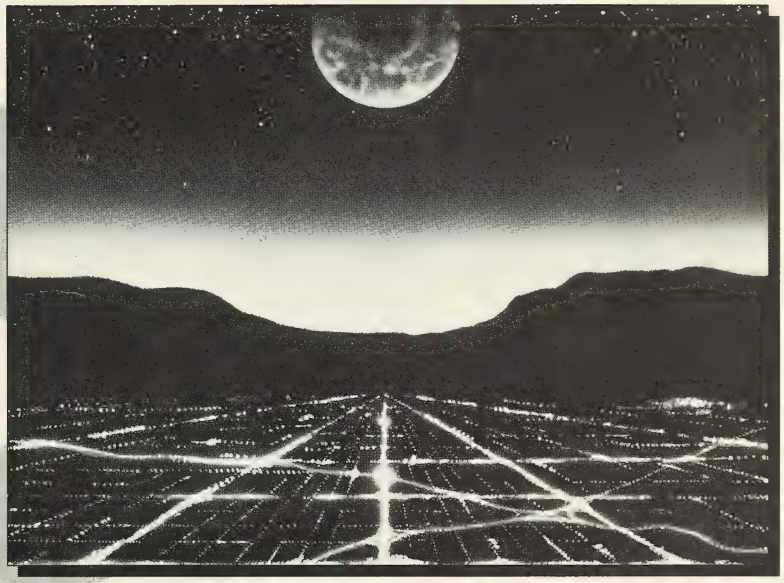




Topic 4

__25__ marks

Mathematics 13



UNIT 4

Coordinate Geometry & Graphing

Distance Learning

Alberta EDUCATION

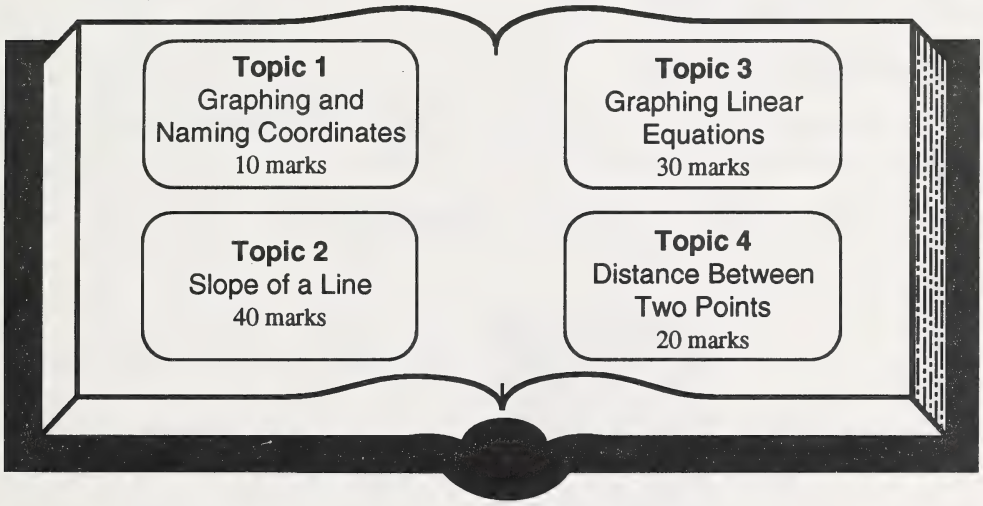
Learning Facilitator's Manual

Unit 4 Overview

This unit called Coordinate Geometry and Graphing is worth 18% of the course work.

This unit consists of four topics. Each topic has an assignment found in the Assignment Booklet.

Unit 4 Coordinate Geometry and Graphing

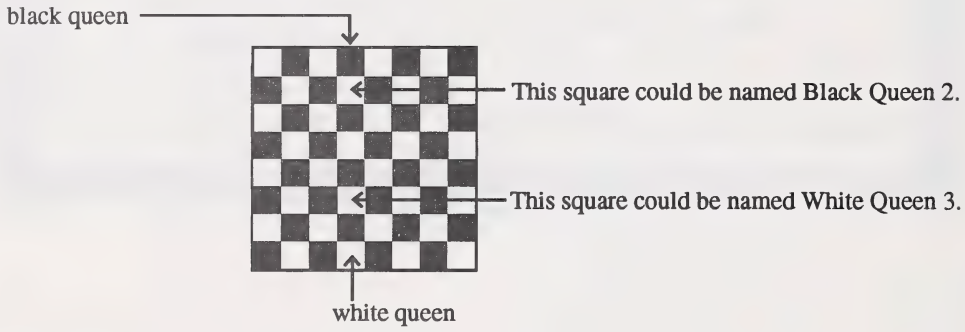


Concept Introduction Possibilities

- Show concretely when order is important. For example, order is significant when getting dressed, packing your groceries at a store, loading a moving van, etc.
- Show where a line will go if it is extended on a graph.
- Show how the slope changes with a change in vertical and/or horizontal distance.
- Show how the Pythagorean theorem is used to find the length of a hypotenuse and how it can also be applied to finding the distance between two points.

Possible Presentation Activities

- Make several ramps with the same hypotenuse lengths but with different rises and runs to show how increasing the slope will increase the speed of a marble rolled down the ramp.
- Use a chess gameboard to show how locations or squares may be named.



- Use maps and charts.

Marking Guidelines

- If the answer is correct, give full marks whether or not any work is given.
- If the answer is incorrect, give partial marks for any correct work shown.

Topic 1: Graphing and Naming Coordinates

(1) 1. a. Circle the correct response. In which quadrant is the point $P(-2, 4)$ located?

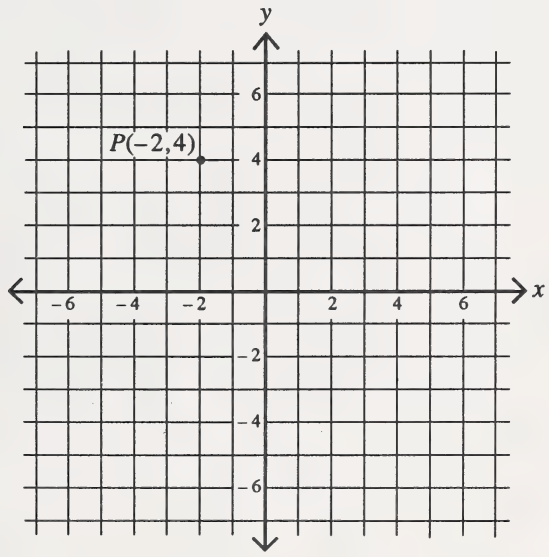
A. Quadrant I

(B.) Quadrant II

C. Quadrant III

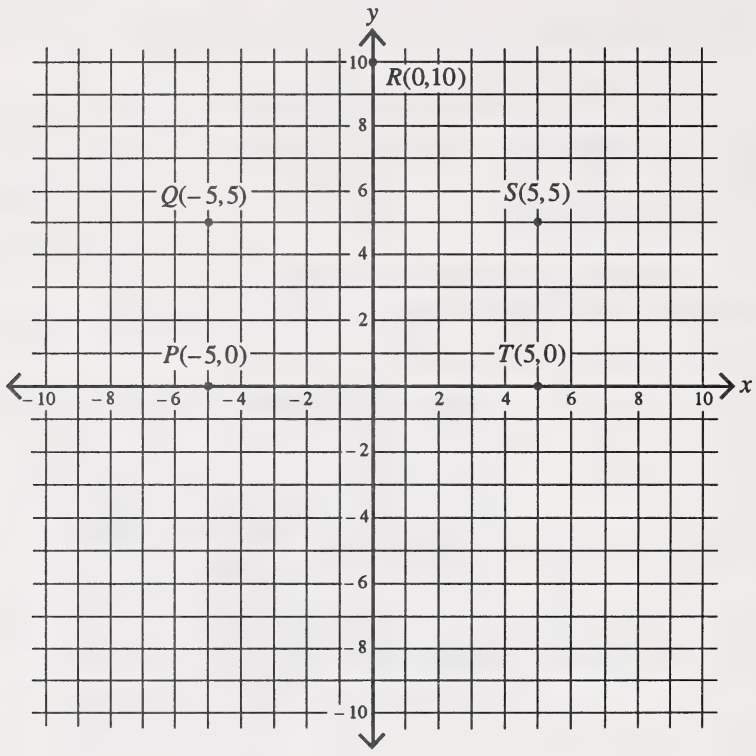
D. Quadrant IV

(1) b. Plot $P(-2, 4)$ on the coordinate plane given.

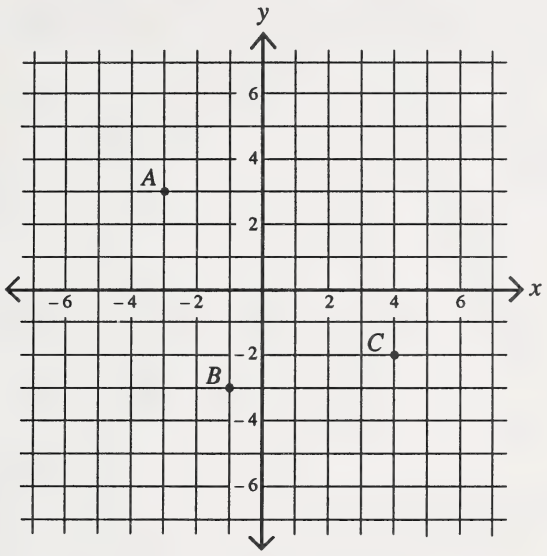


(5) 2. Plot each of the following points on the given coordinate plane.

$P(-5,0)$, $Q(-5,5)$, $R(0,10)$, $S(5,5)$, $T(5,0)$



(3) 3. What are the ordered pairs corresponding to the points A, B, and C?



$A(-3, 3)$

$B(-1, -3)$

$C(4, -2)$

Topic 1

__10__ marks

Topic 2: Slope of a Line

(3) 1. Two points on a coordinate plane are $P(-4,-7)$ and $Q(5,-1)$. What is the slope of $\overline{PQ}$? You may use the given coordinate plane to help you.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

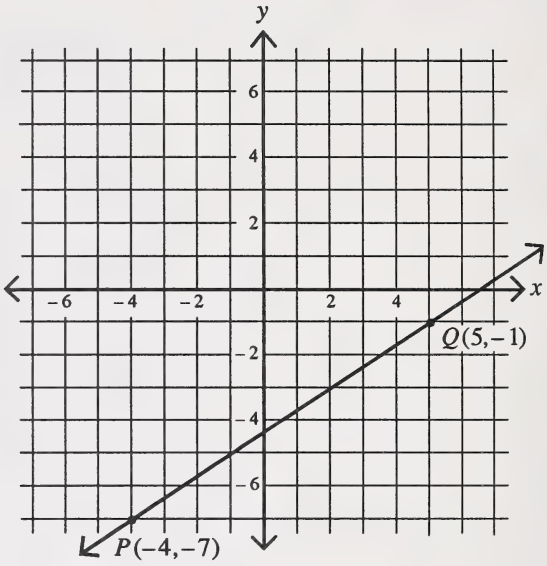
$$m = \frac{-1-(-7)}{5-(-4)}$$

$$= \frac{-1+7}{5+4}$$

$$= \frac{6}{9}$$

$$= \frac{2}{3}$$

The slope of $\overline{PQ}$ is $\frac{2}{3}$.



(3) 2. If a line passes through $A(-3,-1)$ and $B(5,-1)$, what is its slope? You may use the given coordinate plane to help you.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

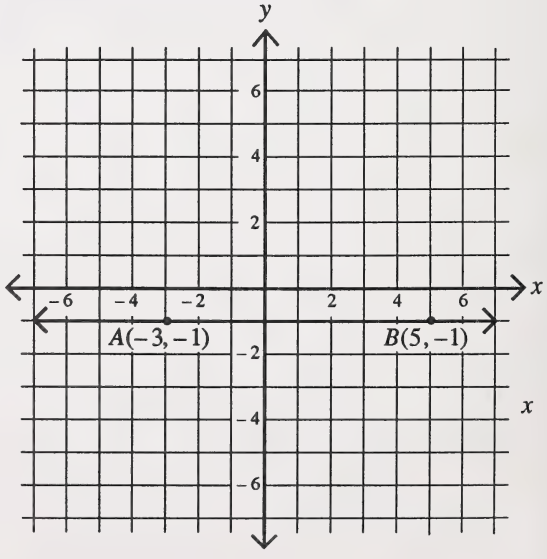
$$m = \frac{-1-(-1)}{5-(-3)}$$

$$= \frac{-1+1}{8+3}$$

$$= \frac{0}{11}$$

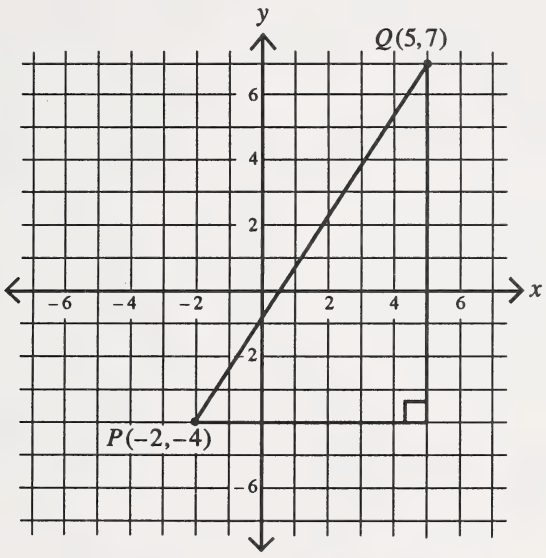
$$= 0$$

All horizontal lines have a zero slope.



③

3. A line segment has endpoints $P(-2,-4)$ and $Q(5,7)$. What is the rise of the line segment PQ? What is the run of the line segment PQ? You may use the given coordinate plane to help you.



$$\text{rise} = 7-(-4)$$
$$= 7+4$$
$$= 11$$

$$\text{run} = 5-(-2)$$
$$= 5+2$$
$$= 7$$

The rise and the run can also be found by counting squares from the right angle in the right triangle shown.

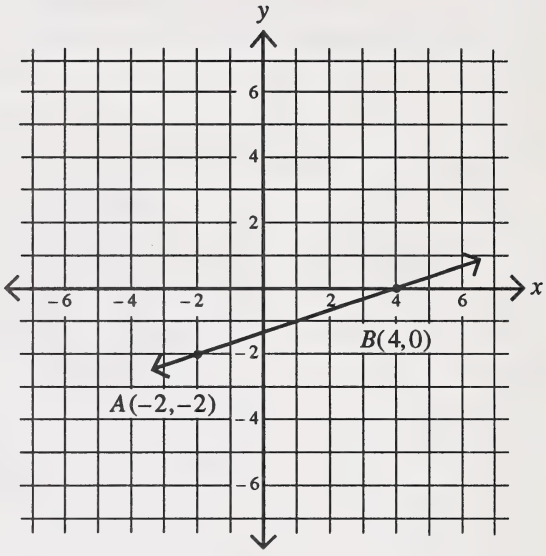
(2) 4. Circle the correct response. If a line has a positive slope, which of the following is true?

A. It is undefined.

B. It is equal to zero.

(C.) It rises to the right.

D. It falls to the right.



Example:

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{2}{6}$$
$$= \frac{1}{3}$$

Note: Only the circled response is required for full marks.

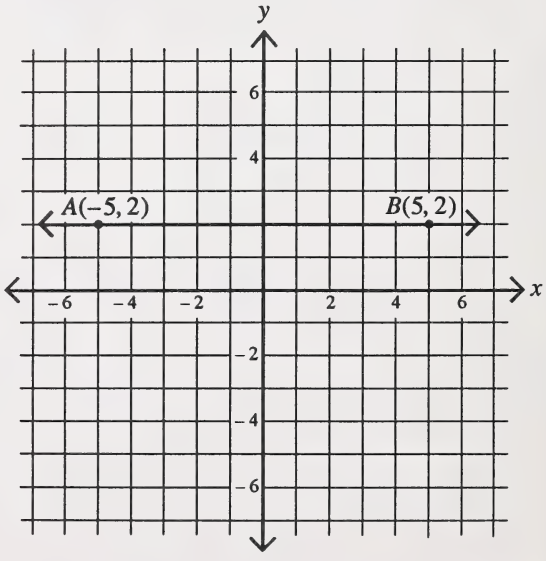
(2) 5. Circle the correct response. If the slope of a line is zero, then the rise is

A. an integer

B. negative

C. equal to the run

(D.) zero



Example:

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{0}{10}$$
$$= 0$$

Note: Only the circled response is required for full marks.

(3) 6. What is the slope of the line $6x-5y=11$?

$$6x-5y=11$$
$$-5y=-6x+11$$
$$\frac{-5y}{-5}=\frac{-6x}{-5}+\frac{11}{-5}$$
$$y=\frac{6x}{5}-\frac{11}{5}$$
$$y=\frac{6}{5}x-\frac{11}{5}$$

The slope of the line $6x-5y=11$ is $\frac{6}{5}$.

(4) 7. Find the slope of the line that is perpendicular to a line that passes through the points $A(2,8)$ and $B(-3,5)$.

$$\text{Slope}=\frac{y_2-y_1}{x_2-x_1}$$

$$m_1=\frac{5-8}{-3-2} \quad \text{or} \quad m_1=\frac{8-5}{2-(-3)}$$
$$=\frac{-3}{-5} \qquad =\frac{3}{2+3}$$
$$=\frac{3}{5} \qquad =\frac{3}{5}$$

$$m_1m_2=-1$$
$$\therefore \frac{3}{5}m_2=-1$$
$$m_2=-1\div\frac{3}{5}$$
$$=-1\times\frac{5}{3}$$
$$=\frac{-5}{3}$$

The slope of the perpendicular line is $\frac{-5}{3}$.

Notice that $\frac{3}{5}$ and $\frac{-5}{3}$ are negative reciprocals of each other.

(4) 8. The run of a ramp is 3 m. The slope is 0.25. What is the rise?

Let x represent the rise.

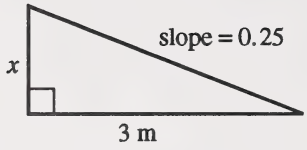
$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$0.25 = \frac{x}{3}$$

$$\frac{0.25}{1} = \frac{x}{3}$$

$$x = 3 \times 0.25$$

$$= 0.75$$



The rise of the ramp is 0.75 m or $\frac{3}{4}$ m.

(1) 9. a. Circle the correct response. The slope of a vertical line is

A. zero

(B.) undefined

C. negative

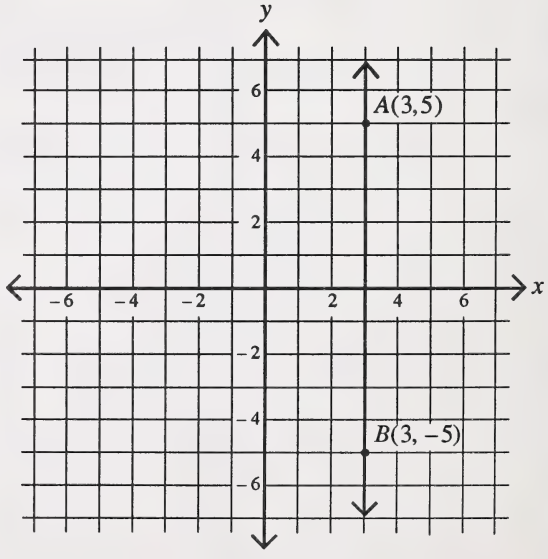
D. positive

Example:

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{10}{0} \text{ (undefined)}$$

Note: Only the circled response is required for one mark.

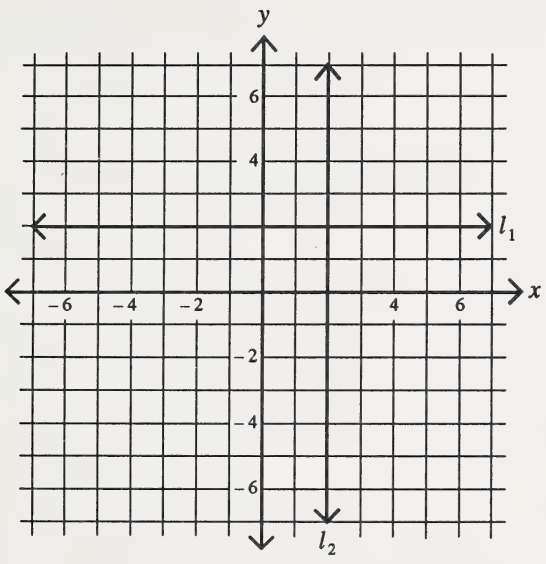


(2) b. Why is this true?

A vertical line has a run of zero. The slope is undefined because it is not possible to divide by zero.

(2) 10. If line l_1 is perpendicular to line l_2, then $m_1 m_2 = -1$. When is this statement not true?

This is not true when l_1 is parallel to the x-axis and l_2 is parallel to the y-axis.



Example:

slope of $l_1 = 0$

slope of l_2 = undefined

Zero multiplied by an undefined slope is meaningless, not -1.

Note: Only the statement is required for full marks.

(3) 11. What is the slope of the line $3x + 4y - 12 = 0$?

$$3x + 4y - 12 = 0$$
$$4y = -3x + 12$$
$$\frac{4y}{4} = \frac{-3x}{4} + \frac{12}{4}$$
$$y = \frac{-3x}{4} + 3$$
$$y = \frac{-3}{4}x + 3$$

$$m = -\frac{3}{4}$$

The slope of $3x + 4y - 12 = 0$ is $\frac{-3}{4}$ or $-\frac{3}{4}$.

Use the following information to answer questions 12, 13, and 14.

The equation of l_1 is $3x+4y-12=0$.
The equation of l_2 is $3y-4x=12$.
The equation of l_3 is $-3x-4y+18=0$.

(3) 12. Give the slopes of l_1, l_2, and l_3.

In slope y-intercept form the equations are as follows:

- The equation of l_1 is $y=\frac{-3}{4}x+3$.
- The equation of l_2 is $y=\frac{4}{3}x+4$.
- The equation of l_3 is $y=\frac{-3}{4}x+4\frac{1}{2}$.

$$m_1 = \text{slope of } l_1 = \frac{-3}{4}$$
$$m_2 = \text{slope of } l_2 = \frac{4}{3}$$
$$m_3 = \text{slope of } l_3 = \frac{-3}{4}$$

(3) 13. Which lines are perpendicular? Explain your answer.

$l_1 \perp l_2$
$l_2 \perp l_3$

$$m_1 m_2 = \frac{-3}{4} \times \frac{4}{3}$$
$$= -1$$

$$m_2 m_3 = \frac{4}{3} \times \frac{-3}{4}$$
$$= -1$$

If the product of the slopes is -1, the lines are perpendicular.

(2) 14. Which lines are parallel? Explain your answer.

$l_1 \parallel l_3$

$m_1 = \frac{-3}{4}$

$m_3 = \frac{-3}{4}$

If the slopes are equal, the two lines are parallel.

Topic 2

__40__ marks

Topic 3: Graphing Linear Equations

(3) 1. Find the y-intercept of the line $6x-5y=11$.

Let $x=0$.

$$6x-5y=11$$
$$6(0)-5y=11$$
$$0-5y=11$$
$$-5y=11$$
$$\frac{-5y}{-5}=\frac{11}{-5}$$
$$y=\frac{-11}{5}$$

or

$$6x-5y=11$$
$$-5y=-6x+11$$
$$\frac{-5y}{-5}=\frac{-6x}{-5}+\frac{11}{-5}$$
$$y=\frac{6}{5}x-\frac{11}{5}$$

↑ y-intercept

The y-intercept is $\frac{-11}{5}$.

(2) 2. a. What is the y-intercept of the line $3x+4y-12=0$?

Let $x=0$.

$$3x+4y-12=0$$
$$3(0)+4y-12=0$$
$$0+4y-12=0$$
$$4y=12$$
$$\frac{4y}{4}=\frac{12}{4}$$
$$y=3$$

or

$$3x+4y-12=0$$
$$4y=-3x+12$$
$$\frac{4y}{4}=\frac{-3x}{4}+\frac{12}{4}$$
$$y=\frac{-3}{4}x+3$$

↑ y-intercept

The y-intercept is 3.

(2) b. What is the x-intercept of the line $3x+4y-12=0$?

Let $y=0$.

$$\begin{aligned} 3x+4y-12&=0\\ 3x+4(0)-12&=0\\ 3x+0-12&=0\\ 3x-12&=0\\ 3x&=12\\ \frac{3x}{3}&=\frac{12}{3}\\ x&=4 \end{aligned}$$

The x-intercept is 4.

(3) 3. Write the equation $3x-2y-4=0$ in slope y-intercept form.

$$\begin{aligned} 3x-2y-4&=0\\ -2y&=-3x+4\\ \frac{-2y}{-2}&=\frac{-3x}{-2}+\frac{4}{-2}\\ y&=\frac{3x}{2}-2\\ y&=\frac{3}{2}x-2 \end{aligned}$$

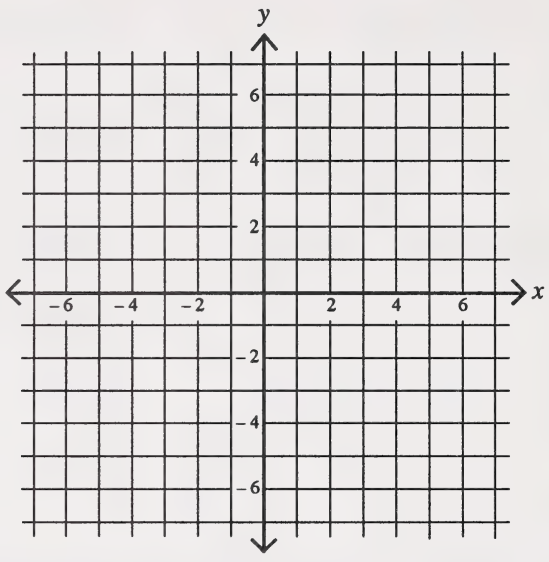
(1) 4. Circle the correct response. The equation for the y-axis is

(A.) $x=0$

B. $x+1=0$

C. $y=0$

D. $x-1=0$



All points on the y-axis have a first coordinate of 0.

Note: Only the circled response is required for full marks.

(4) 5. Fill in the following table of values for the equation $y=-4x+8$.

x	-3	-2	-1	0	1	2	3	4
y	**20**	**16**	**12**	**8**	**4**	**0**	**−4**	**−8**

(3) 6. Circle the correct response. Which two lines pass through the point $(-4, 5)$? You may use the given coordinate plane to help you.

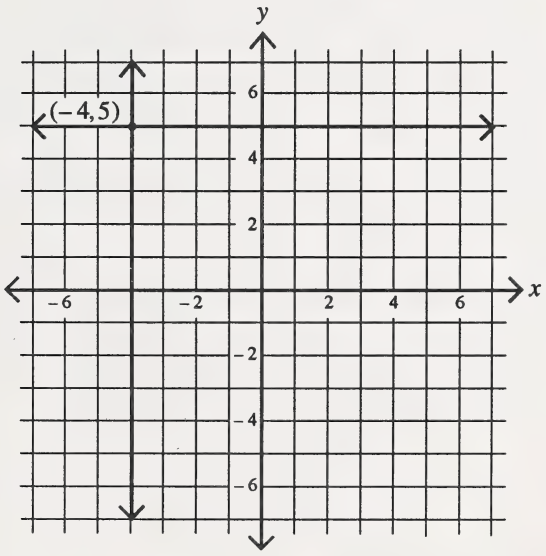
A. $x-4=0$ and $y+5=0$

(B.) $x+4=0$ and $y-5=0$

C. $y-5=0$ and $x-4=0$

D. $y+5=0$ and $x+4=0$

E. none of the above



The lines $x+4=0$ and $y-5=0$ pass through the point $(-4, 5)$.
Note that $x+4=0$ can also be written as $x=-4$, and $y-5=0$ can also be written as $y=5$.

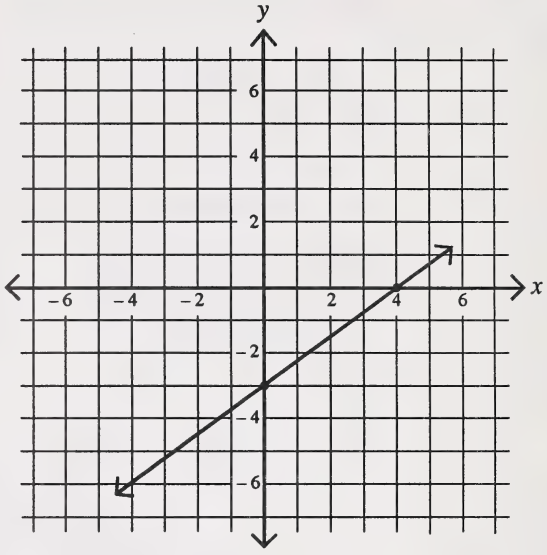
Note: Only the circled response is required for full marks.

④ 7. Draw the graph of the line $3x-4y=12$. Be sure to show the x- and y-intercepts.

Method 1

$$3x-4y=12$$
$$-4y=-3x+12$$
$$y=\frac{-3x}{-4}+\frac{12}{-4}$$
$$y=\frac{3x}{4}-3$$
$$y=\frac{3}{4}x-3 \leftarrow y\text{-intercept}$$
$$\uparrow$$
$$\text{slope}$$

$$\text{slope}=\frac{\text{rise}}{\text{run}}$$
$$=\frac{3}{4}$$



Plot the y-intercept $(0,-3)$ and then use the slope of $\frac{3}{4}$ to find the second point.

Method 2

x	-3	-2	-1	0	1	2	3	4
y	$-5\frac{1}{4}$	$-4\frac{1}{2}$	$-3\frac{3}{4}$	-3	$-2\frac{1}{4}$	$-1\frac{1}{2}$	$\frac{-3}{4}$	0

Method 3

Find the x- and y-intercepts.

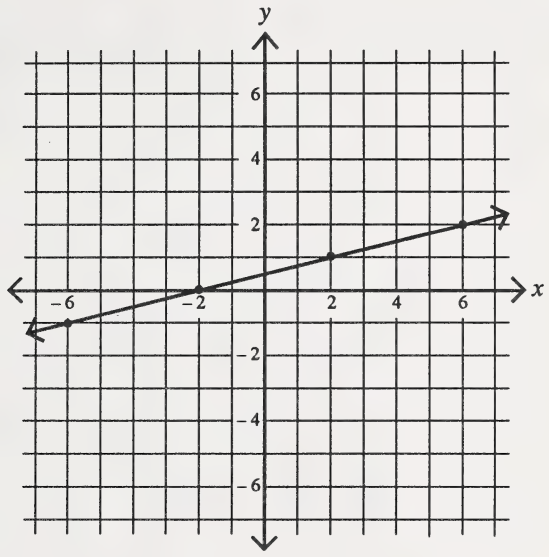
To find the x-intercept, let $y=0$.

$$3x-4y=12$$
$$3x-4(0)=12$$
$$3x-0=12$$
$$3x=12$$
$$\frac{3x}{3}=\frac{12}{3}$$
$$x=4$$

To find the y-intercept, let $x=0$.

$$3x-4y=12$$
$$3(0)-4y=12$$
$$0-4y=12$$
$$-4y=12$$
$$\frac{-4y}{-4}=\frac{12}{-4}$$
$$y=-3$$

(4) 8. Give the slope and the x- and y-intercepts for the line on the following coordinate plane.

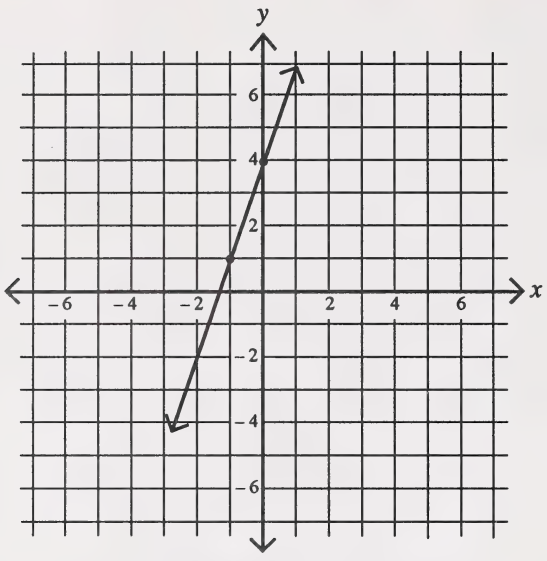


$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{2}{8}$$
$$= \frac{1}{4}$$

x-intercept $= -2$

y-intercept $= \frac{1}{2}$

(4) 9. Draw the graph of the line which crosses the y-axis at 4 and has a slope of 3.



Step 1: Plot the point where the line crosses the y-axis.

Step 2: Use the formula for slope.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\frac{3}{1} = \frac{\text{rise}}{\text{run}}$$

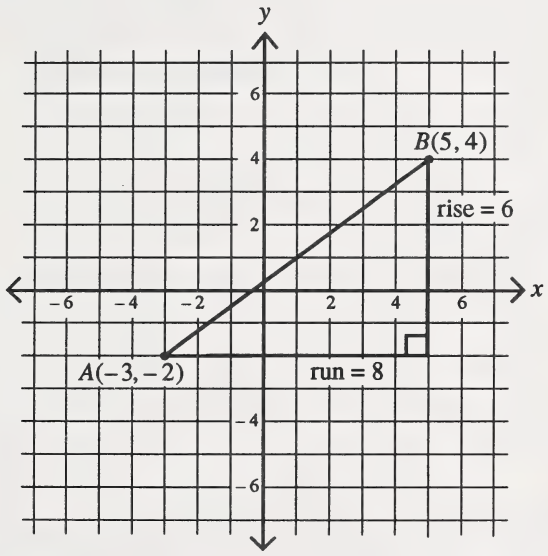
Go down three units and to the left one unit to get the second point.

Topic 3

__30__ marks

Topic 4: Distance Between Two Points

④ 1. Find the length of the line segment that joins the two points represented by $A(-3,-2)$ and $B(5,4)$.



$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{[5-(-3)]^2+[4-(-2)]^2}$$
$$=\sqrt{(5+3)^2+(4+2)^2}$$
$$=\sqrt{8^2+6^2}$$
$$=\sqrt{64+36}$$
$$=\sqrt{100}$$
$$=10$$

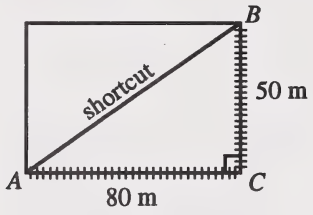
or

$$c^2=a^2+b^2$$
$$c^2=8^2+6^2$$
$$c^2=64+36$$
$$c^2=100$$
$$c^2=\sqrt{100}$$
$$c=10$$

The length of the line segment is 10 units.

Note: The graph is not required in the student response.

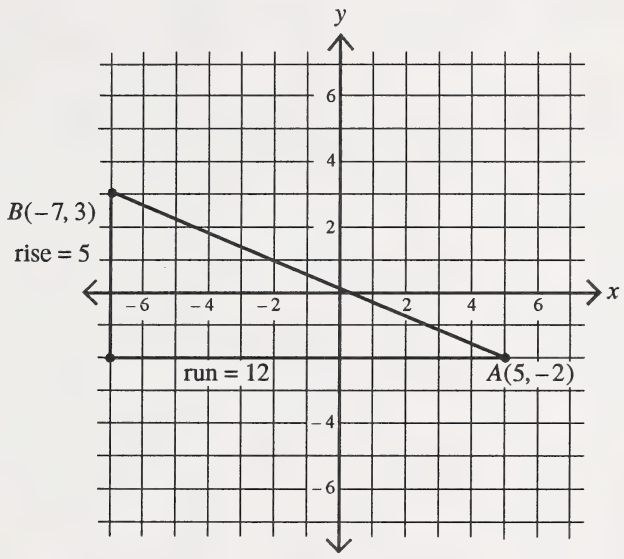
④ 2. Find the distance that Chad saves by taking the shortcut across the field shown in the diagram. Round your answer to two decimal places.



$$c^2 = a^2 + b^2$$
$$c^2 = 80^2 + 50^2$$
$$c^2 = 6400 + 2500$$
$$c^2 = 8900$$
$$\sqrt{c^2} = \sqrt{8900}$$
$$c = 94.339\,811\,32$$
$$= 94.34 \text{ m}$$

Going from A to C to B is $80 + 50 = 130$ m.
The distance Chad saves by taking the shortcut is $130 - 94.34 = 35.66$ m.

⑥ 3. Use the Pythagorean theorem and the distance formula to find the distance between the points represented by $A(5,-2)$ and $B(-7,3)$.



$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{[5-(-7)]^2+(-2-3)^2}$$
$$=\sqrt{(5+7)^2+(-5)^2}$$
$$=\sqrt{(12)^2+(-5)^2}$$
$$=\sqrt{144+25}$$
$$=\sqrt{169}$$
$$=13$$

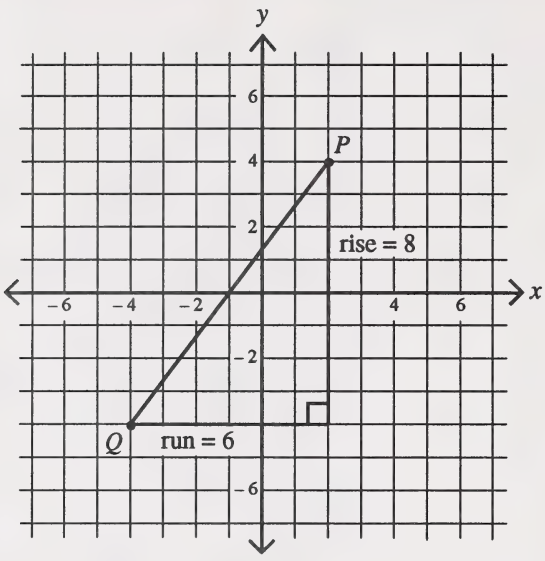
or

$$c^2=a^2+b^2$$
$$c^2=12^2+5^2$$
$$c^2=144+25$$
$$c^2=169$$
$$c=\sqrt{169}$$
$$=13$$

The distance between point A and point B is 13 units.

Note: The graph is not required in the student response.

④ 4. Find the slope and the length of line segment PQ. Get your information from the coordinate plane provided.



$$\text{Slope of } \overline{PQ} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{8}{6}$$
$$= \frac{4}{3}$$

The slope of $\overline{PQ}$ is $\frac{4}{3}$.

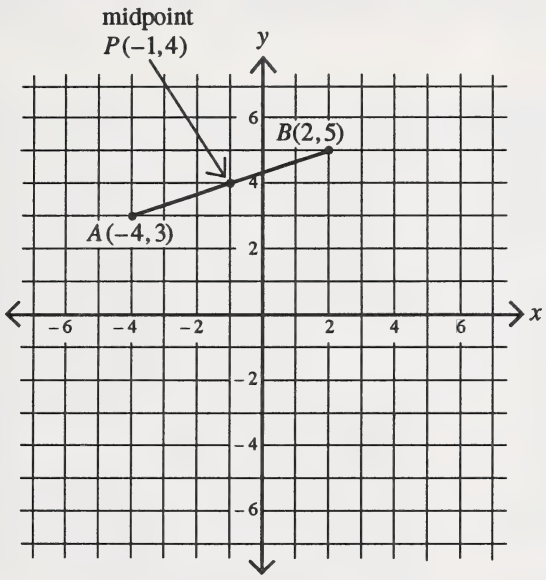
$$\text{Length of } \overline{PQ} = \sqrt{6^2 + 8^2}$$
$$= \sqrt{36+64}$$
$$= \sqrt{100}$$
$$= 10 \text{ units}$$

or $P(2,4)$ and $Q(-4,-4)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-4-2)^2 + (-4-4)^2}$$
$$= \sqrt{(-6)^2 + (-8)^2}$$
$$= \sqrt{36+64}$$
$$= \sqrt{100}$$
$$= 10 \text{ units}$$

The length of $\overline{PQ}$ is 10 units.

(2) 5. Find the midpoint of the line segment joining $A(-4, 3)$ and $B(2, 5)$. Use the following coordinate plane to check your answer.



$$x = \frac{x_1 + x_2}{2} \qquad y = \frac{y_1 + y_2}{2}$$

$$= \frac{-4+2}{2} \qquad = \frac{3+5}{2}$$

$$= \frac{-2}{2} \qquad = \frac{8}{2}$$

$$= -1 \qquad = 4$$

The midpoint of $\overline{AB}$ is $P(-1, 4)$.

Topic 4

20 marks

Mathematics 13

UNIT 5
Relations

Distance Learning

Alberta EDUCATION

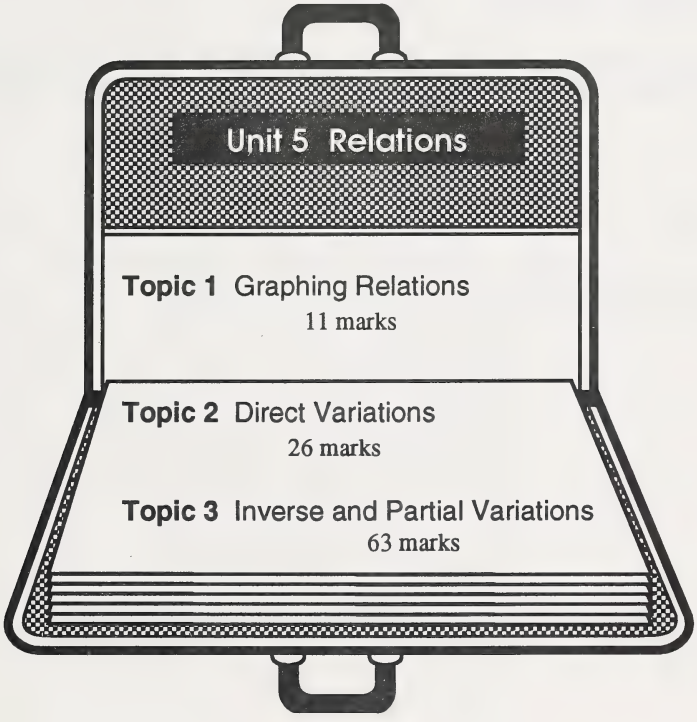
Learning Facilitator's Manual

Cover photo: WESTFILE INC.

Unit 5 Overview

This unit called Relations is worth 12% of the course work.

The unit consists of three topics. Each topic has an assignment found in the Assignment Booklet.



Concept Introduction Possibilities

- Distinguish between increasing and decreasing quantities.
- Show how one activity is dependent on another activity.
- Compare direct variation to inverse and partial variation.
- Show how direct variation is similar to proportion.

Possible Presentation Activities

- Use concrete materials to show how one activity changes as the other changes. For example, demonstrate how moving a light bulb closer to the base of an object increases the length of the object's shadow.

- Use graph paper.

Marking Guidelines

- If the answer is correct, give full marks whether or not any work is provided.
- If the answer is incorrect, give partial marks for any correct work shown.

Topic 1: Graphing Relations

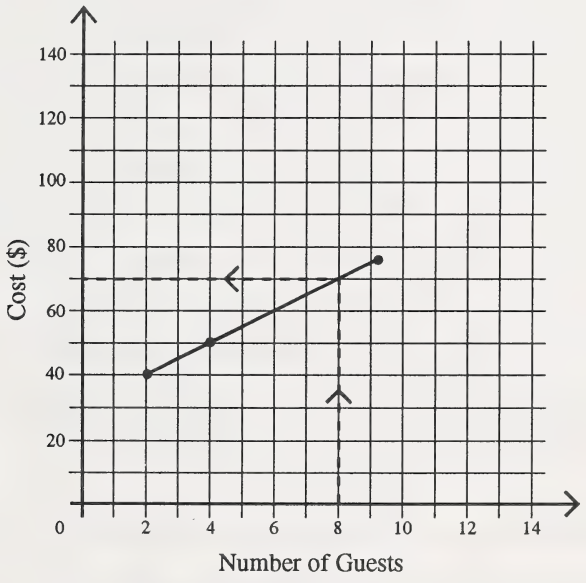
1

1. From the following statement, determine which is the dependent variable. The distance (d) a car travels in an hour depends on its speed (s).

   Distance is the dependent variable.

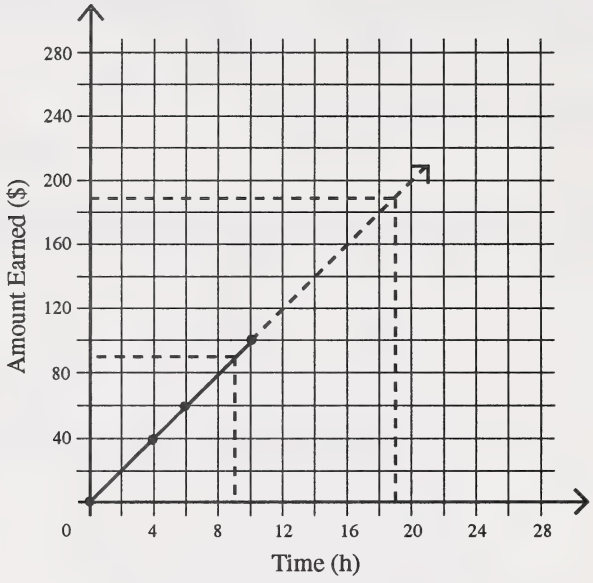
2

2. From the following graph, determine the cost when the number of guests is 8.



The cost for eight guests would be $70.

3. Use the following graph to answer the questions.



(2) a. Extrapolate to determine how much was earned in 19 h.

From extrapolation, $190 was the amount earned in 19 h.

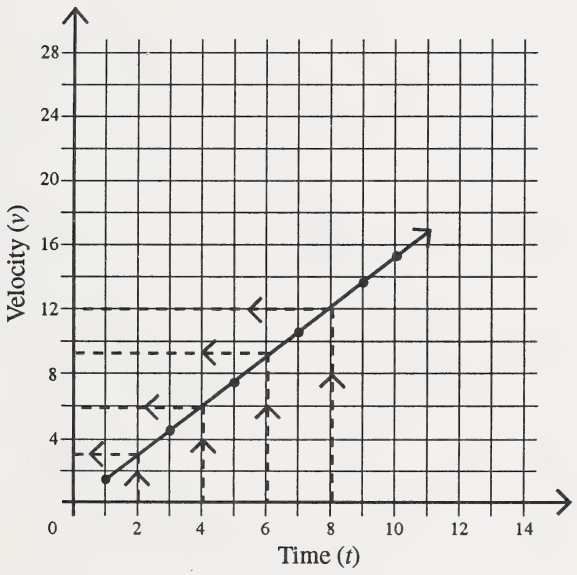
(1) b. Interpolate to determine how much was earned in 9 h.

From interpolation, $90 was the amount earned in 9 h.

(2) 4. a. Use the method of interpolation to complete the following table.

Time (t)	1	2	3	4	5	6	7	8	9	10
Velocity (v)	1.5	**3**	4.5	**6**	7.5	**9**	10.5	**12**	13.5	15

(2) b. Graph the information from the table of values on the following axes.



(1) c. The independent variable is **time** .

Note that the independent variable is on the horizontal axis and the dependent variable is on the vertical axis.

Topic 1

11 marks

Topic 2: Direct Variations

(2) 1. In the equation $y = 4.5x$, what is the constant of variation?

The constant of variation (k) equals 4.5.

(4) 2. Note that s varies directly as t varies. When $s = 15$ and $t = 2.5$, what is the constant of variation? Show your work for full marks.

$$s \propto t \qquad \text{or} \qquad s = kt$$

$$s = k \times t \qquad \frac{s}{t} = k$$

$$15 = k \times 2.5 \qquad \frac{15}{2.5} = k$$

$$k \times 2.5 = 15 \qquad k = \frac{15}{2.5}$$

$$\frac{k \times 2.5}{2.5} = \frac{15}{2.5} \qquad = \frac{150}{25}$$

$$k = \frac{150}{25} \qquad = 6$$

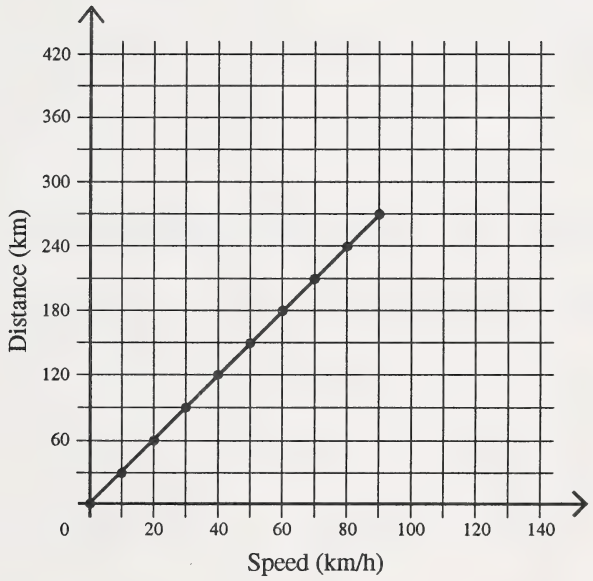
$$= 6$$

The constant of variation is 6.

(5) 3. The following table shows the distance (d) that a car would travel in three hours at various speeds (s) in kilometres per hour.

Speed (s) (km/h)	0	10	20	30	40	50	60	70	80	90
Distance (d) (km)	0	30	60	90	120	150	180	210	240	270

Use this information to make a graph.



(2) 4. Circle the correct response. Which of the following represents a direct variation?

A. $y = \frac{8}{x}$

(B.) $y = 20x$

C. $y = 16 + 20x$

D. $yx = 20$

The answer is B.

(4) 5. The area (A) of a figure varies directly as the height (h) varies. The base is fixed. When $A = 153$, $h = 17$. Write an equation relating the area (A) to the height (h).

$$A \propto h$$
$$A = kh$$
$$153 = k \times 17$$
$$k \times 17 = 153$$
$$\frac{k \times 17}{17} = \frac{153}{17}$$
$$k = 9$$

Therefore, the equation is $A = 9h$ or $h = \frac{1}{9}A$.

6. The distance (d) varies directly as the time (t) varies.

(2) a. Determine an equation relating distance (d) to time (t). Use $d = 52$ m when $t = 13$ s.

$$d \propto t$$
$$d = kt$$
$$52 = k(13)$$
$$k = \frac{52}{13}$$
$$= 4$$
$$\therefore d = 4t$$

(2) b. Find d when $t = 58.5$ s.

$$d = 4t$$
$$= 4(58.5)$$
$$= 234 \text{ m}$$

(3) 7. Circle the correct response or responses. Which of the following describes direct variation?

(A.) The larger the diameter is, the bigger the circle will be.

(B.) The more you pump air into a tire, the larger the tire gets.

(C.) The more you study, the higher the marks you receive.

D. The larger amount of money you spend, the less money you save.

E. The older you get, the less hair you have.

F. The greater the flow of the river, the less time it takes to canoe from point *A* to point *B*.

Answers A, B, and C describe direct variation.

(2) 8. Circle the correct response or responses. Which of the following describes $d \propto t$?

(A.) d varies as t varies.

(B.) d varies directly as t varies.

(C.) d is proportional to t.

(D.) d is directly proportional to t.

Answers A, B, C, and D describe $d \propto t$.

Topic 2

__26__ marks

Topic 3: Inverse and Partial Variations

(2) 1. Circle the correct response. Which of the following is an inverse relation?

A. $y = kx$

B. $y = 20 + 12x$

(C.) $xy = 20$

D. $\frac{y}{x} = k$

The answer is C.

(3) 2. Note that y varies inversely as x varies. In a certain situation $x = 7$ when $y = 13$. Find the constant of variation.

$$y \propto \frac{1}{x}$$
$$xy = k$$
$$k = xy$$
$$= 13 \times 7$$
$$= 91$$

The constant of variation is 91.

3. Study the following table, and answer the questions.

Time (t) (s)	2	4	5	8	10	12.5	20	25	50
Speed (s) (m/s)	50	25	20	12.5	10	8	5	4	2

(2) a. What type of variation does this table describe?

The table describes an inverse variation.

(3) b. What is the constant of variation?

$$t \propto \frac{1}{s}$$
$$ts = k$$
or $k = ts$

Choose any ordered pair from the table of values. Use $t = 2$ s and $s = 50$ m/s.

$$ts = k$$
$$2 \times 50 = k$$
$$k = 100$$

The constant of variation is 100.

(3) c. What is the equation that produced this table?

$ts = k$

$ts = 100$

Note that $t = \frac{100}{s}$ and $s = \frac{100}{t}$ would also be correct.

(2) 4. You are given the following facts.

- Ground-level temperature is 20°C.
- The temperature at the bottom of a well 500 m deep is 29°C.
- The temperature in a coal mine 1.5 km below the surface is 39°C.

What method can you use to find the temperature of the earth's crust 10 km down?

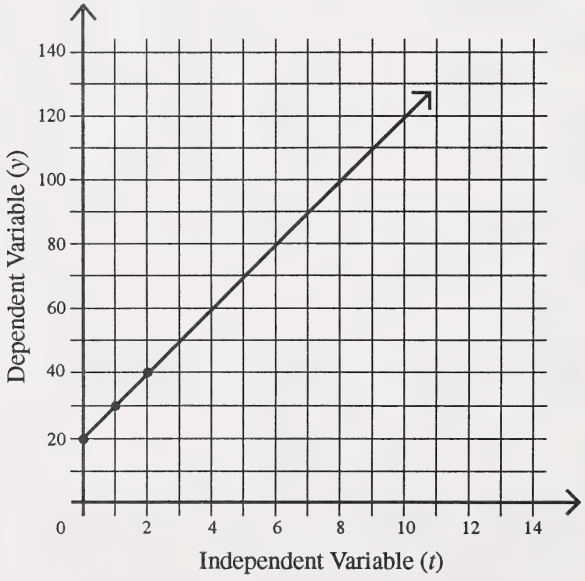
Extrapolation can be used.

5. You are given the equation $y = a + kt$, where $a = 20$ and $k = 10$.

(3) a. Complete the table of values.

t	0	1	2
y	**20**	**30**	**40**

(3) b. Graph the equation.



③ 6. Circle the correct response. If y varies inversely as x varies and $x = 4$ when $y = 2$, then which of the following is the variation equation?

A. $y = 2x$

B. $y = 8x$

C. $\frac{y}{x} = 4$

(D.) $y = \frac{1}{2}x$

The answer is D.

$y \propto x$

$y = kx$

$k = \frac{y}{x}$

$= \frac{2}{4}$

$= \frac{1}{2}$

$\therefore y = \frac{1}{2}x$ from $y = kx$

(3) 7. The cost (C) of supplying snacks and punch at a party is related to the number of guests (N) who attend. There is a fixed cost of $20 as well. The following table shows a breakdown of the total cost to the number of guests attending.

Number of Guests	0	10	20	30	40	50	60	70
Cost ($)	20	70	120	170	220	270	320	370

Write an equation to show this relation.

To find k in $C = F + kn$, choose any ordered pair from the table of values. Choose $n = 10$ and $C = \$70$.

$$C = F + kn$$
$$70 = 20 + k \times 10$$
$$10k + 20 = 70$$
$$10k = 50$$
$$\frac{10k}{10} = \frac{50}{10}$$
$$k = 5$$

The equation showing this relation is $C = F + 5n$.

(2) 8. Circle the correct response. Which one of the following refers to a partial variation?

A. $y = kx$

(B.) $y = 20 + 12x$

C. $xy = 20$

D. $\frac{y}{x} = k$

The answer is B.

9. A mechanic charges a fixed rate of $25 for every job, plus $38 per hour.

(2) a. Write an equation to express the mechanic's total charge (C) in relation to time (t).

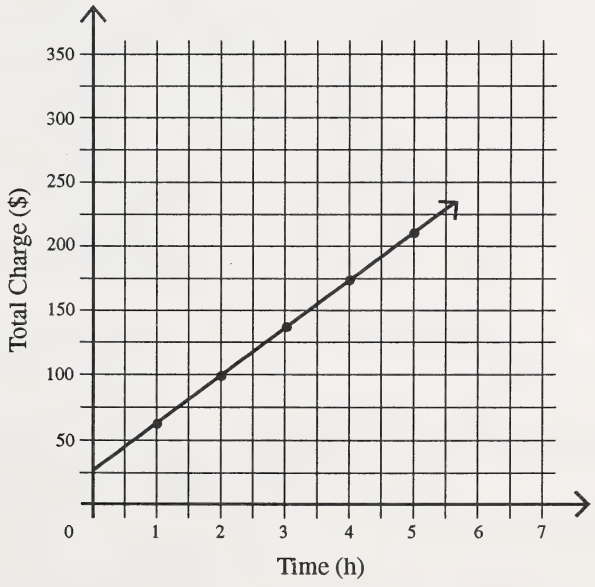
$C = F + kt$

$C = 25 + 38t$

(2) b. Complete the table of values.

Time (t) (h)	1	2	3	4	5
Total charge (C) ($)	**63**	**101**	**139**	**177**	**215**

(2) c. Graph the relation.



(4) 10. A van load of people took the ferry from Nanaimo to Vancouver. The fee structure was \$40.75 for each van plus x dollars per person. If there were fifteen people in the van and the total charge was \$97, what was the charge per person?

$$C = F + kx$$
$$97 = 40.75 + 15x$$
$$56.25 = 15x$$
$$3.75 = x$$

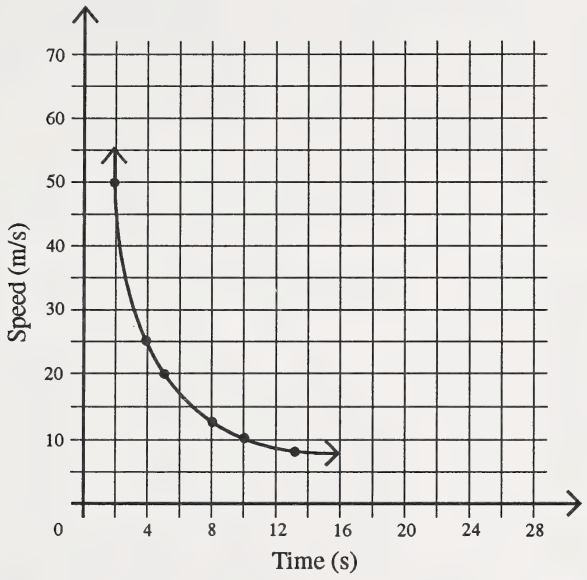
The charge per person was \$3.75.

(3) 11. If $s \propto \frac{1}{t}$, what is the value of k if $s = 6$ and $t = 4$?

$$s \propto \frac{1}{t}$$
$$s = \frac{k}{t}$$
$$k = st$$
$$= 6 \times 4$$
$$= 24$$

The value of k is 24.

12. Use the graph to answer the following questions.



(2) a. What is the independent variable?

Time in seconds is the independent variable.

(3) b. What kind of variation is this curve describing?

This curve represents an inverse variation.

(4) c. According to this curve, what is the constant of variation?

When $t = 10$ s, $s = 10$ m/s.

You can choose 10 s and 10 m/s from the point on the graph, or you may choose any other point to obtain your values for s and t.

$$s \propto \frac{1}{t}$$

$$s = \frac{k}{t}$$

$$k = st$$

$$= 10 \times 10$$

$$= 100$$

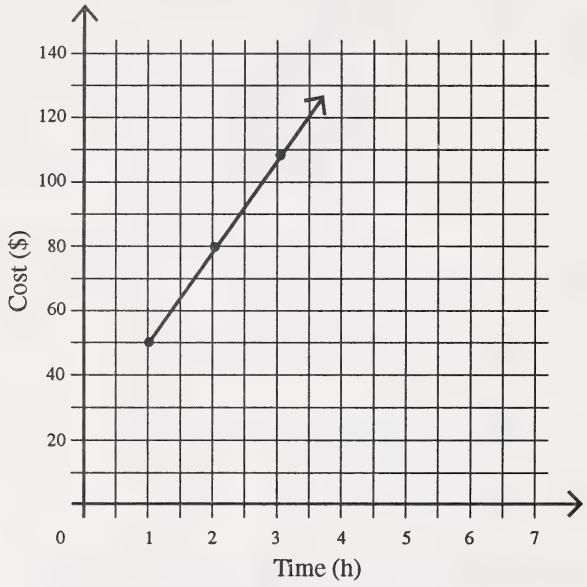
The constant of variation is 100.

(3) 13. The following table shows one variable increasing while the other is decreasing. Why is it not an inverse variation?

H	24	21	18	15	12	9	6	3
t	1	2	3	4	5	6	7	8

There is no constant of variation. The product of each ordered pair is not the same.

14. Use the graph to answer the following questions.



(2) a. What is the dependent variable?

The cost (C) is the dependent variable.

(1) b. Does the graph pass through the origin?

No, the graph does not pass through the origin.

(2) c. What type of variation is the graph describing?

This graph describes partial variation.

④ 15. When a small pulley and a large pulley are connected, the small pulley will revolve more rapidly than the large pulley. Suppose the large pulley has a diameter of 40 cm and revolves at 45 RPM. Find the number of RPM of the small pulley if its diameter is 10 cm.

$$d \propto \frac{1}{s}$$
$$d = \frac{k}{s}$$
$$k = ds$$
$$= 40 \times 45$$
$$= 1800$$

Use $k = 1800$ and $d = 10$ in $k = ds$.

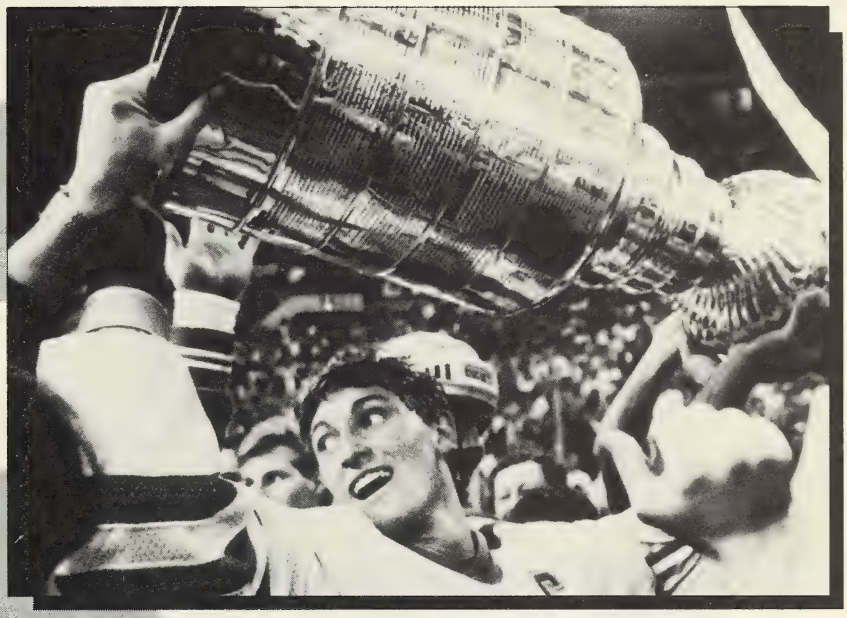
$$k = ds$$
$$1800 = 10 \times s$$
$$\frac{10 \times s}{10} = \frac{1800}{10}$$
$$s = 180$$

The small pulley revolves at 180 RPM.

Topic 3

__63__ marks

Mathematics 13



UNIT 6

Statistics

Distance Learning

Alberta EDUCATION

Learning Facilitator's Manual

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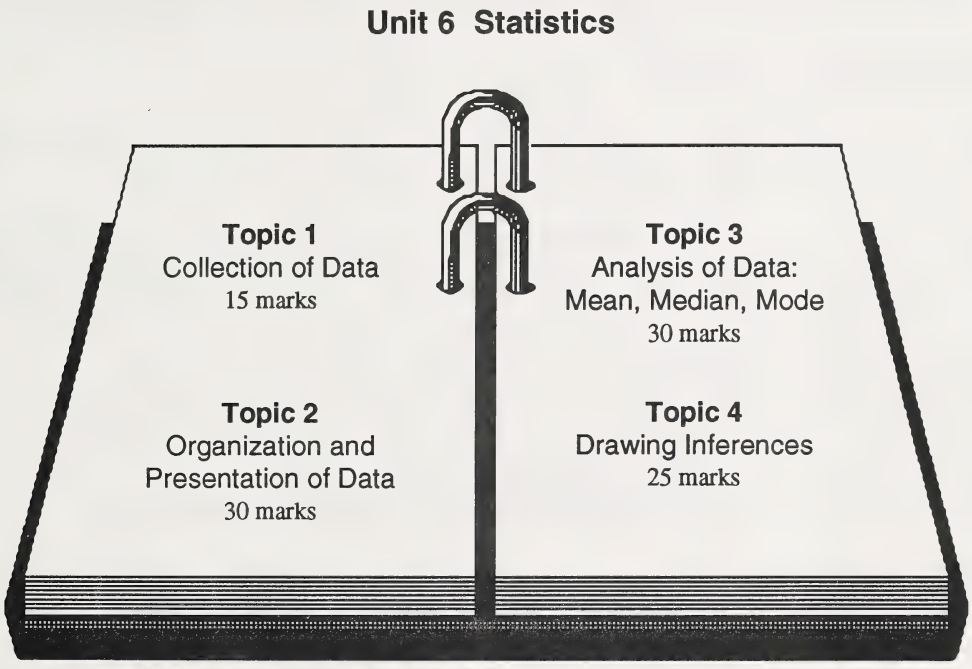
Unit 6 Overview

This unit called Statistics is worth 14% of the course mark.

The unit consists of four topics. Each topic has an assignment found in the Assignment Booklet.



Concept Introduction Possibilities

- Discuss sampling methods used for such surveys as TV polls and political polls.
- Discuss how a sample may be biased.
- Carry out a classroom survey such as asking students how many hours of TV they watched the previous night. Discuss whether or not the results could be applied to the whole school, the town or city, or the whole country.

Possible Presentation Activities

- Collect graphs and charts from magazines and newspapers. Have students present them to the class. They should try to answer questions regarding sample size, method of selection, and possible bias.
- Give the students a set of topics for a survey. See page 102 of the Teacher Resource Manual for some possibilities. Then the Students, individually or in groups of two or three, can choose a topic, select a sampling method, determine the size of the sample, and carry out the survey. They can then organize and display the data using one of the methods discussed. They can later present their findings to the class and answer questions from their classmates and the teacher.

Marking Guidelines

- If the answer is correct, give full marks whether or not any work is shown.
- If the answer is incorrect and work is shown, give partial marks for any correct work shown.

Topic 1: Collection of Data

(1)

1. For the following question, circle the correct response.

   Imagine that you want to find the average weight of a Canadian high school student. You decide to weigh every high school student in all of Calgary, Regina, and Kelowna. The **sample** of your population is

   A. every high school student in Canada

   B. any students in Calgary, Regina, and Kelowna

   C. all the students in Canada

   (D.) every high school student in Calgary, Regina, and Kelowna

(2)

2. Suppose there is to be a federal election and a poll is taken to determine which political party is favoured by the people of Canada. It is decided that 1000 people from each of the thirty most populated cities across the country will be selected at random from the phone book and asked which political party they will support in the coming election.

   Is this considered a **biased** sample? Explain your answer.

   This is a biased sample because it does not include areas with smaller populations. The sample should include people from towns, villages, and rural areas.

(1) 3. Suppose there is to be a provincial election and a survey is taken to determine which political party is favoured by the people of Alberta. It is decided that every third person shopping at a local store in Red Deer, Alberta is to be asked which political party they will support in the coming election.

Is this an appropriate type of sample to determine which political party is favoured in Alberta? Explain your answer.

No, this is not an appropriate type of sample because people in other areas of the province are not part of the sample.

(1) 4. For the following question, circle the correct response. Explain your answer.

Suppose you want to determine which TV program is watched most often in the province of British Columbia. Which sampling method will work best to help you determine this?

A questionnaire is sent randomly to people selected from

A. the town of Osoyoos

(B.) various provincial cities

C. the city of Victoria

D. Vancouver Island

This is the best choice because you want to determine which TV program is watched most often in the whole province, not in just one area of the province.

(1) 5. Suppose there is to be a provincial election and a survey is taken to determine which political party is favoured by the people of British Columbia. It is decided that members of a steelworkers union will be randomly selected and asked which political party they will support in the coming election.

Is this an appropriate type of sample to determine which political party is favoured in British Columbia? Give a reason for your answer.

No, this is not an appropriate type of sample because members of a steelworkers union might be partial toward one party or another. For example, while they were on strike, they might have been forced back to work by a certain political party. A survey designed to determine what the whole population favours needs to sample all parts of the population.

(1) 6. Suppose you need to find the average amount of money a family living in Edmonton spends on food in one month. To solve this problem, which sampling method will most accurately predict the average amount of money spent?

Collect food receipts from all the

(A.) grocery stores

B. fast-food restaurants

C. gasoline service stations

D. expensive restaurants

(4) 7. Suppose you were asked to collect enough data to determine the average shoe size of forty randomly selected people. Explain where, when, and how you conducted the survey.

Answers may vary. An example is shown as follows.

Ask at least forty people to give their first name and their shoe size, and record their responses.

Choose a shoe store or the shoe department of a department store as the location for the survey. Interview people in the ladies', mens' and childrens' store departments.

Choose a Friday evening or a Saturday to ensure a more representative sample of the population. If people refuse to answer, interview more people until you get forty names and responses.

(4) 8. Explain how to use a random number table to pick any four days from all of the days in a year.

Let the numbers 000 to 364 represent the days.

Pick a spot in the random number table to start. If the three digits are from 000 to 364, then you have one day selected. If the digits are from 365 to 999, then no day has been selected.

Go to the next group of three digits. Be sure to follow in order rather than jump around the table.

Continue these steps until you have four numbers between 000 and 365. These numbers will represent the days. For example, 020 would be the twentieth day, 109 would be the one hundred and ninth day, and so on.

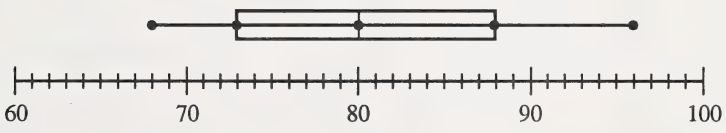
Topic 1

__15__ marks

Topic 2: Organization and Presentation of Data

(5) 1. The total number of points scored by the Taber Turtlebacks in a twenty-four-game season is presented as follows. Construct a box-and-whisker plot and analyse the data by stating the median, the quartiles, and the extremes.

68	70	70	70	72	[72]
[74]	74	74	76	76	(80)
(80)	82	84	86	86	[88]
[88]	88	92	94	96	96



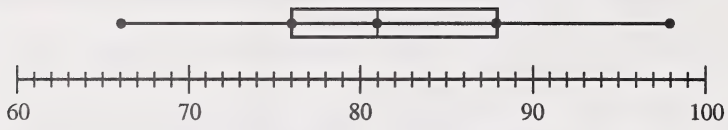
$$\text{Lower quartile} = \frac{72+74}{2}$$
$$= \frac{146}{2}$$
$$= 73$$

$$\text{Upper quartile} = \frac{88+88}{2}$$
$$= \frac{176}{2}$$
$$= 88$$

$$\text{Median} = \frac{80+80}{2}$$
$$= \frac{160}{2}$$
$$= 80$$

The extremes are 68 and 96.

(3) 2. The total number of points scored by the Wainwright Waisters in a forty-five-game season is presented by the box-and-whisker plot that follows.



a. What percentage of the population is included in the box?

The percentage of the population included in the box is 50%.

b. All of the values are between **66** and **98**.

c. What is the median of these values?

The median is 81.

④ 3. A basketball team scored 89 points in a single game. The top four players on the team are shown with their respective number of points for the game.

Player	Number of Points
Duncan D.	16
Jaybar A.	8
Birdie B.	17
Johnson M.	18

Circle the correct response. Given the preceding data, determine which graphing technique would **best visually represent** the data. Then, draw the graph using this technique and be sure to explain the advantage of using this particular type of representation.

(A.) pictograph

B. bar graph

C. circle graph or pie chart

D. stem-and-leaf plot

Points Scored in a Basketball Game

Legend: Each ● represents two points.

Pictographs are used for making comparisons of similar data.

Note: A bar graph would be acceptable as well.

(3) 4. Suppose a survey of several Canadian homes revealed the following numbers of boxes of breakfast cereal consumed each week. Organize this data into a frequency table.

8	9	2	2	1
0	8	2	6	2
4	1	8	8	5
8	2	6	2	8
8	8	5	7	0
9	1	3	0	1
4	8	0	6	9
8	0	7	2	1

Number of Boxes of Cereal Consumed per Week

Number of Boxes	Tally	Frequency
0	𝍸	5
1	𝍸	5
2	𝍸 ||	7
3	|	1
4	||	2
5	||	2
6	|||	3
7	||	2
8	𝍸 𝍸	10
9	|||	3

③ 5. Draw a pictograph to visually represent the set of data given. You are free to select your own pictograph symbol provided it relates to the nature of the data given.

A survey of forty Canadian households revealed the following number of beds found in each home.

2	4	4	7	7	7	6	7	4	4
4	4	3	4	6	6	7	8	7	5
7	7	6	6	3	4	2	2	1	1
3	1	4	2	6	7	6	7	7	6

Frequency Table

Number of Beds	Tally	Frequency
1	III	3
2	IIII	4
3	III	3
4	𝍸 IIII	9
5	I	1
6	𝍸 III	8
7	𝍸 𝍸 I	11
8	I	1

Number of Beds per Canadian Household

1 bed

2 beds

3 beds

4 beds

5 beds

6 beds

7 beds

8 beds

Legend: represents two households

③ 6. A computer has randomly generated the following even numbers. Organize the data into a stem-and-leaf plot.

72	90	60	36	50
88	72	76	36	86
66	40	76	82	48
42	62	50	44	50
30	82	40	46	36
36	72	90	72	54
30	62	90	38	90
68	90	48	86	70

Correct Order

Stem	Leaf
3	0 0 6 6 6 6 8
4	0 0 2 4 6 8 8
5	0 0 0 4
6	0 2 2 6 8
7	0 2 2 2 2 6 6
8	2 2 6 6 8
9	0 0 0 0 0

Another Example of Correct Order

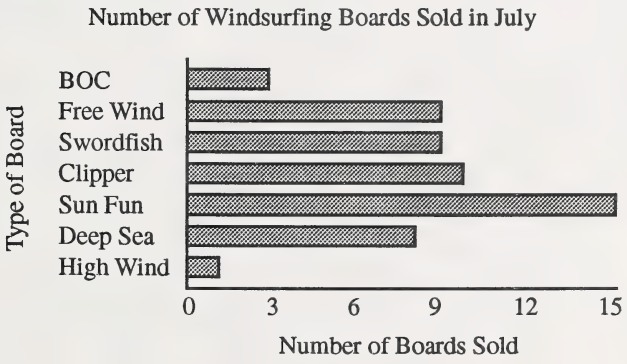
Stem	Leaf
3	0 6 0 6 6 8 6
4	2 0 0 8 4 6 8
5	0 0 0 4
6	6 8 2 2 0
7	2 2 2 6 6 2 0
8	8 2 2 6 6
9	0 0 0 0 0

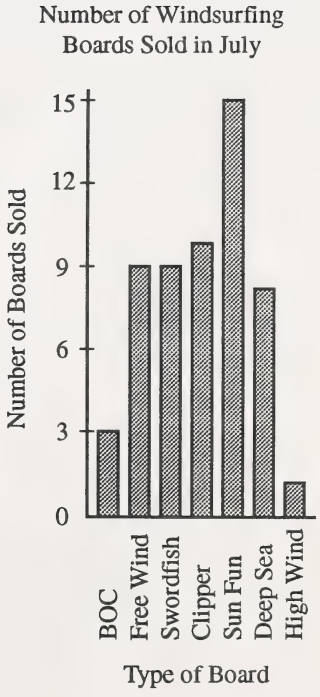
④ 7. Using a bar graph, accurately represent the data given.

Several stores in California that sell sporting goods were surveyed to determine how many windsurfing boards were sold during July 1991. The results of this survey are shown in the following table.

Type of Board	Frequency
BOC	3
Free Wind	9
Swordfish	9
Clipper	10
Sun Fun	15
Deep Sea	8
High Wind	1

The graph may be drawn horizontally or vertically.





(5) 8. The top four goal scorers on a local hockey team are shown with their respective number of goals for the season. A total of 167 goals were scored by the entire team all season.

All of the other players scored a total of 44 goals.

Complete the table and draw a circle graph to accurately represent the data. Be sure your graph is properly labelled and proportioned.

Player	Number of Goals	Percentage (rounded to the nearest whole number)	Degrees (rounded to the nearest whole number)
Tina	31	**19%**	**68°**
Arthur	26	**16%**	**58°**
Ashley	31	**19%**	**68°**
Monique	35	**21%**	**76°**
Others	44	**26%**	**94°**
	Totals	**101%**	**364°**

Top Goal Scorers

Topic 2

Due to rounding, the percentages total 101% which results in an excess of 4° since the degree measures total 364°.

__30__ marks

Topic 3: Analysis of Data: Mean, Median, Mode

(1) 1. For the following question, circle the correct response.

In shoe sales, the person responsible for ordering the number of new shoes is interested in knowing which size is selling the best. Which of the following shows this?

A. mean

(B.) mode

C. median

D. range

(2) 2. Determine the mode for the following data.

11	13	9	16	9	6	14	12	7
10	12	8	8	14	15	11	14	16

Rearrange the data in ascending order. Although this is not necessary, it makes it easier to find the mode.

6	7	8	8	9	9	10	11	11
12	12	13	14	14	14	15	16	16

The mode is 14.

(2) 3. A group of six numbers has an arithmetic mean of 8. If each of the six numbers is increased by 7, what is the new arithmetic mean?

$$\text{Mean} = \frac{6\times 8+6\times 7}{6}$$
$$= \frac{48+42}{6}$$
$$= \frac{90}{6}$$
$$= 15$$

The new arithmetic means is 15.

(2) 4. Determine the range of the given data.

46.50	45.50	46.10	45.30	46.30	46.70	47.90
46.00	47.00	47.40	47.50	46.40	47.50	47.30

Rearrange the data in ascending order.

45.30	45.50	46.00	46.10	46.30	46.40	46.50
46.70	47.00	47.30	47.40	47.50	47.50	47.90

$\text{Range} = 47.90 - 45.30$

$= 2.60$

The range is 2.60.

(2) 5. Determine the median of the following data.

5	4	14	1	8	10	14	11	18
17	10	14	10	18	18	8	16	

1, 4, 5, 8, 8, 10, 10, 10, $\boxed{11}$, 14, 14, 14, 16, 17, 18, 18, 18

The median is 11.

(3) 6. Determine the mean of the following data. If necessary, express your answer to two decimal places.

−6	1	5	−7	−5	−10	−7
−5	1	5	7	−6	−10	7

$$\text{Mean} = \frac{-6+1+5+(-7)+(-5)+(-10)+(-7)+(-5)+1+5+7+(-6)+(-10)+7}{14}$$

$$= \frac{-30}{14}$$

$$\doteq -2.142\,857\,143$$

$$\doteq -2.14$$

The mean is -2.14 rounded to two decimal places.

(1) 7. Complete the following statement with the correct term.

The middle term in a list of numbers arranged in ascending order is called the **median** .

(2) 8. A group of eight numbers has a mode of 25. If each of the eight numbers is decreased by 9, what is the new mode of the group of numbers?

Mode $= 25 - 9$
$= 16$

The new mode is 16.

(1) 9. Complete the following statement using the correct term or terms.

The mode and the **mean or median** are measures of central tendency.

(2) 10. Determine the mode of the following data.

28	30	26	33	26	23	31	29	24
27	29	25	25	31	32	28	31	33

23, 24, 25, 25, 26, 26, 27, 28, 28, 29, 29, 30, [31, 31, 31,] 32, 33, 33

The mode is 31.

(3) 11. Determine the mean of the following data. If necessary, express your answer accurate to two decimal places.

10	−3	9	−7	5	−7	−7
5	−3	9	4	10	−7	4

$$\text{Mean} = \frac{10+(-3)+9+(-7)+5+(-7)+(-7)+5+(-3)+9+4+10+(-7)+4}{14}$$

$$= \frac{22}{14}$$
$$\doteq 1.571\,428\,571$$
$$\doteq 1.57$$

The mean is 1.57 rounded to two decimal places.

(2) 12. Determine the median of the following data.

95	96	59	84	77	51	70
39	86	58	68	46	89	37

37, 39, 46, 51, 58, 59, [68, 70,] 77, 84, 86, 89, 95, 96

$$\text{Median} = \frac{68+70}{2}$$
$$= \frac{138}{2}$$
$$= 69$$

(4) 13. Determine the mean, the median, the mode, and the range for the data used in the stem-and-leaf diagram shown. Round the mean to one decimal place.

Stem	Leaf
2	5 5 8
3	2 3 4 4 5 9
4	6 7 7
5	4
6	1 [1 3] 8 ← median
7	0 1 3 4 4 5 [8 8 8] ← mode
8	2 3 5 5

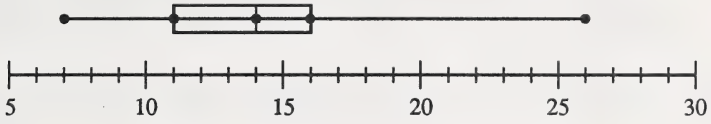
$$\text{Mean} = \frac{\text{sum of all elements}}{30}$$
$$= \frac{1738}{30}$$
$$\doteq 57.933\,333\,33$$
$$\doteq 57.9$$

$$\text{Median} = \frac{61+63}{2}$$
$$= \frac{124}{2}$$
$$= 62$$

$$\text{Mode} = 78$$

$$\text{Range} = 85 - 25$$
$$= 60$$

(3) 14. Using the box-and-whisker plot shown, determine the median, the range, and the lower quartile for the data.



Median = 14

Range = 26 – 7
= 19

Lower quartile = 11

Topic 3

30 marks

Topic 4: Drawing Inferences

⑥ 1. An office manager has two secretaries but can only afford to keep one of them. The manager decides to keep a record of their individual typing speeds, in words per minute, and uses this to make a decision.

Over a period of two weeks, the manager collected the following data.

Secretary *A*:

51	48	52	57	54
53	57	54	56	60

Secretary *B*:

47	52	54	60	42
60	50	59	73	70

If you were the office manager, determine which of the secretaries you would keep. To do this, construct a back-to-back stem-and-leaf diagram comparing the two groups of data. Also determine the median for each secretary. Based on your graph and the median, indicate which secretary will remain employed. Be sure to explain your reasons for your choice of either Secretary *A* or Secretary *B*.

Typing Speed

Secretary *A*		Secretary *B*
	8 | 4	2 7
1 2 3 ④ ④ 6 7	7 | 5	0 2 ④ ⑨
	0 | 6	0 0
	7	0 3

$$\text{Median} = \frac{54+54}{2} = \frac{108}{2} = 54$$

$$\text{Median} = \frac{54+59}{2} = \frac{113}{2} = 56.5$$

The median for Secretary *A* is 54.

The median for Secretary *B* is 56.5.

Based on this information, Secretary *B* should remain employed. Secretary *B* is also showing more improvement.

(3) 2. Fill in the blanks.

a. The probability of tossing a coin and getting heads or tails is **1**.

b. The probability of picking a green marble from a jar containing red marbles is **0**.

c. The probability of picking a card with the number 3 on it, from a deck of 52 cards, is $\frac{4}{52} = \frac{1}{13}$.

(3) 3. A survey asked 2000 people what their favourite TV show is. The results are as follows.

Bobsy Show	33%
57 Minutes	27%
Resigning Women	17%
Bash	13%
Other	10%

In a city of 500 000 people, how many viewers would you predict will be watching the Bobsy Show this week?

$$\frac{33}{100} \times 500\,000 = 165\,000$$

You would predict that 165 000 people will be watching the Bobsy Show this week.

(2) 4. For the following question, answer **true** or **false**.

During the evening news, it is announced that the probability it will rain tomorrow is 45%. Someone listening to the broadcast could then conclude that the probability it will not rain tomorrow is $P(no\ rain) = \frac{11}{20}$.

The conclusion is **true**.

(2) 5. For the following question, circle the correct response.

The probability that an event will occur may be represented by the ratio of two numbers. Determine the missing term (?) from the following ratio.

$$? = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

A. outcomes

(B.) probability

C. population

D. sample

(3) 6. A sewing bag contains 56 yellow spools and 40 white spools of thread. If you randomly select a spool from the bag, what is the probability that the spool will **not** be white?

$$P(\textit{not white}) = \frac{56}{96}$$
$$= \frac{7}{12} \text{ or } 0.58\dot{3}$$

The probability that the spool of thread will not be white is 7 out of 12 or $0.58\dot{3}$.

(3) 7. The probability of rolling a sum of 3 with two dice is $P(\textit{sum of 3}) = \frac{1}{18}$.

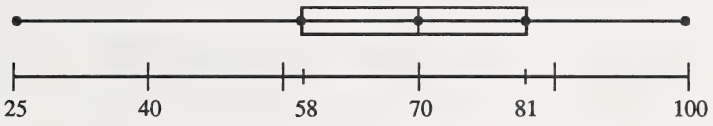
Given that a pair of dice is rolled 72 times, how many sums of 3 would you expect to occur?

$$\frac{\text{one sum of 3}}{\text{18 rolls}} = \frac{x}{\text{72 rolls}}$$
$$x = 72 \div 18$$
$$= 4$$

You would expect four sums of 3 to occur.

③ 8. The following box-and-whisker plot shows the distribution of marks on a recent mathematics exam.

What is the probability that the mark for a student chosen at random is between 58% and 70%?



Note that 58% is the lower quartile mark and
70% is the median, so 25% or $\frac{1}{4}$ of all marks are in this area.

$\therefore P(58\% \text{ to } 70\%) = \frac{1}{4}$ or 25%

The probability that a student's mark is between 58% and 70% is 1 in 4.

Topic 4

__25__ marks

Final Test

There are two copies of the final test: the student's copy which is perforated and designed for photocopying and possible faxing and the teacher's copy which includes a marking guide.

Note:

The student's copy of this final test should be kept secure by the teacher. Students should not have access to this test until it is assigned in a supervised situation. The answers should be stored securely and retained by the teacher at all times.

Students should be reminded to bring a mathematical set that has a compass, protractor, and ruler to the test.

MATHEMATICS 13

FINAL TEST

GENERAL INSTRUCTIONS

1. The time limit for this test is **two hours**.

2. This test consists of two parts. Complete both parts.

3. Write all answers legibly in pen or pencil.

4. Calculators that are not programmed may be used.

5. For written response questions, be sure to show all your work. Part marks will be given for partially correct solutions.

6. It is better to complete all the work you know well first. After that you can deal with the difficult ones. Do not spend too much time on any one problem. If you finish the test before the two-hour time limit, go back and check your answers.

TOTAL MARKS: 125

PART A: Multiple Choice – 50 marks

PART B: Written Response – 75 marks

TEACHER'S COPY

Value

PART A: MULTIPLE CHOICE

50

In each of the questions, indicate the correct answer by placing the appropriate letter A, B, C, or D in the blank provided.

Example:

_B_ The value of 2^3 is

A. 6

B. 8

C. -8

D. -6

_D_ 1. In $-8, -6, 2, 10, 10, 14, 34$, the number 10 is referred to as the

A. range

B. average

C. mean

D. mode

_A_ 2. The range for $-5, -3, 0, 4, 9, 10,$ and 13 is

A. 18

B. 8

C. 13

D. 5

TEACHER'S COPY

C 3. Two straight lines are parallel. If the slope of one of these lines is -5, the slope of the other line is

A. 5

B. $\frac{1}{5}$

C. -5

D. $-\frac{1}{5}$

A 4. What is the degree of the given polynomial?
$-6a^3b^2c+4ab^2c-3$

A. 6

B. 10

C. 3

D. 4

B 5. The mean for $-3, -1, 0, 5, 6,$ and 8 is

A. 0

B. 2.5

C. -4

D. -2.5

B 6. The value for x in the table is

4	6	8	x	15
16	24	32	44	60

A. 10

B. 11

C. 12

D. 14

C 7. According to the number of terms in $\frac{-2ab}{5}$, what is the expression called?

A. quotient

B. binomial

C. monomial

D. trinomial

D 8. An isosceles triangle always has the following.

A. all angles are less than 90°

B. two angles are less than 90° and one angle is greater than 90°

C. all three sides have different lengths

D. at least two sides have equal lengths

A 9. Two straight lines are perpendicular. If the slope of one of these lines is $-\frac{5}{4}$, the slope of the other line is

A. $\frac{4}{5}$

B. -1

C. $-\frac{4}{5}$

D. $\frac{5}{4}$

C 10. The value for y in the table is

-5	-1	0	1	5
10	2	0	-2	y

A. -5

B. 5

C. -10

D. 10

B 11. Which of the following is the square of the hypoteneuse of a right triangle equal to?

A. the square root of the base

B. the sum of the squares of the other two sides

C. the product of the other two sides

D. the square root of the altitude

TEACHER'S COPY

C 12. The simplified form for $(-6a+4b-10c)+(7a-5b+3c)-(-a-5b-4c)$ is

A. $-14a+7b-17c$

B. $4a+2b-3c$

C. $2a+4b-3c$

D. $3a+3c-3b$

A 13. In the diagram, which property makes these two triangles congruent?

A. SSS

B. ASA

C. SAS

D. AAS

D 14. In 14, 18, 18, and 35, 18 can be referred to as the median or the

A. mean

B. bimodal

C. range

D. mode

B 15. In $-4z$, what does z represent?

A. term

B. literal coefficient

C. sum

D. numerical coefficient

A 16. A straight line is perpendicular to the x-axis. What can be said about this slope?

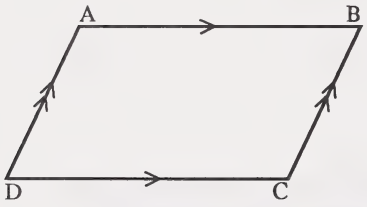
A. The slope is undefined.

B. The slope is positive.

C. The slope is zero.

D. The slope is negative.

D 17. Quadrilateral $ABCD$ is a parallelogram.



$m\angle B + m\angle C = 180°$ because

A. two angles are corresponding angles

B. two angles are complimetary

C. two angles are vertically opposite angles

D. two angles are same-side interior angles

B 18. Which of the following is an integer?

A. $3 \div 0$

B. $-16 \div (-8)$

C. $1 \div (-3)$

D. $4 \div 15$

A 19. The $y = mx + b$ form for $-20 + 5y = 10x$ is

A. $y = 2x + 4$

B. $5y = 10x + 20$

C. $y = 10x - 20$

D. $5y = -20 - 10x$

C 20. Find the value for $\frac{-2a^2bc^2}{30a}$ given $a = -3$, $b = 3$, and $c = -5$.

A. 1350

B. -15

C. 15

D. 5310

A 21. If y varies directly as x and $y = 36$ when $x = 6$, the constant of variation is

A. 6

B. 8

C. 12

D. 2

B 22. In simplest form, $\frac{2^{-3}}{2^{-5}}$ is

A. $\frac{1}{4}$

B. 4

C. $\frac{1}{2}$

D. 2

C 23. The product of $(b-7)(b+4)$ is

A. $b^2+3b+28$

B. $b^2+3b-28$

C. $b^2-3b-28$

D. $b^2+11b+28$

__D__ 24. The numerical coefficient in $-a^4b^2c$ is

A. 1

B. 4

C. 2

D. -1

__B__ 25. The evaluated form for $(-3)^3 \times (-3)^0$ is

A. 9

B. -27

C. 27

D. 0

__A__ 26. Determine an equation relating a to b if a is directly proportional to b, when $b = 3.2$ and $a = 22.4$.

A. $a = 7b$

B. $b = 7a$

C. $7 = ab$

D. $ab = 7$

C 27. How many variables are there in $4a^3c^3x^2+2ac-3x+14abx$?

A. 9

B. 2

C. 4

D. 3

D 28. The quotient for $\frac{16x^3y^6z^4+10x^3y^3-2x^2y^3}{2x^2y^3}$ is

A. $8xyz^4+5x^2$

B. $16x^2y+10x-1$

C. $8xyz+5xy-2xy^2$

D. $8xy^3z^4+5x-1$

A 29. When a line is parallel to the y-axis on a coordinate plane, the slope is

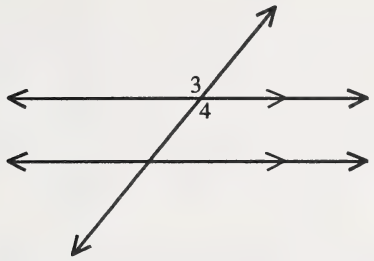
A. undefined

B. negative

C. zero

D. positive

B 30. The special name given to $\angle 3$ and $\angle 4$ in the diagram is



A. corresponding

B. vertically opposite

C. supplementary

D. alternate

B 31. The factored form for $12y^2 - 3y$ is

A. $2y^2(6y-1)$

B. $3y(4y-1)$

C. $2y(6y-1)$

D. $4y(2y-1)$

D 32. The slope of a straight-line graph that falls from left to right is

A. unequal

B. positive

C. equal

D. negative

TEACHER'S COPY

A 33. Which of the following equations show partial variation?

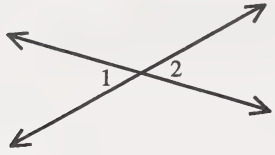
A. $C = m + bx$

B. $C = \frac{x}{y}$

C. $xy = C$

D. $\frac{C}{x} = \frac{1}{y}$

C 34. In the diagram the angles labelled $\angle 1$ and $\angle 2$ are called



A. adjacent angles

B. alternate angles

C. vertically opposite angles

D. corresponding angles

B 35. Which of the following is undefined?

A. $\frac{45}{x}$ where $x = -1$

B. $\frac{45}{x}$ where $x = 0$

C. $\frac{45}{x}$ where $x \neq 0$

D. $\frac{45}{x}$ where $x = 1$

D 36. In a survey, all possible samples belong to the

A. probability

B. random number table

C. data

D. population

TEACHER'S COPY

A 37. The lowest common denominator for $\frac{4}{5}$, $\frac{3}{10x}$, and $\frac{1}{2x^2}$ is

A. $10x^2$

B. $50x^2$

C. $20x^3$

D. $100x$

C 38. In $\overrightarrow{AB}$, the A is called a (an)

A. start

B. line

C. endpoint

D. ray

B 39. Which of the following gives a full word description for $\overline{AB} \perp \overleftrightarrow{CD}$?

A. Ray AB is parallel to line CD.

B. Line segment AB is perpendicular to line CD.

C. Line segment AB is parallel to line CD.

D. Line segment AB is not parallel to line CD.

A 40. The point $F(-4, -1)$ is in

A. Quadrant III

B. Quadrant II

C. Quadrant IV

D. Quadrant I

B 41. If the probability of an event is 1, this is also known as

A. an impossible event

B. a certain event

C. a statistics event

D. a biased event

C 42. When graphing a relation, what does the vertical axis represent?

A. the independent variable

B. the related variable

C. the dependent variable

D. the unrelated variable

A 43. To select a red ball from a bag of white balls is a (an)

A. impossible event

B. possible event

C. uncertain event

D. certain event

C 44. If all four arithmetical operations are found in a number expression, the one which must be done first is

A. multiplication

B. addition

C. division

D. subtraction

A 45. The two areas on a coordinate plane intersect at the

A. origin

B. ordered pair

C. quadrant

D. dependent variable

TEACHER'S COPY

B 46. Simplify this expression.

$$\frac{4x^2y^3}{15z^4} \div \frac{12xy^2}{5z^2}$$

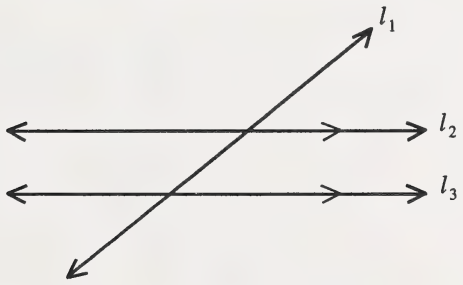
A. $\frac{4x}{5y}$

B. $\frac{xy}{9z^2}$

C. $\frac{3z}{20x^2y}$

D. $\frac{48z^3}{75xy}$

C 47. What is the name of l_1 in this diagram?



A. intersector

B. bisector

C. transversal

D. perpendicular

D 48. What is the minimum number of points needed to make a straight-line graph?

A. one

B. three

C. four

D. two

__A__ 49. Monomials, binomials, and trinomials can be referred to generally as

A. polynomials

B. four-term polynomials

C. terms

D. factors

__A__ 50. Rearrange $-4xy+3x^2-7y^2$ in descending order of powers.

A. $3x^2-4xy-7y^2$

B. $-7y^2+3x^2-4xy$

C. $-7y^2-4xy+3x^2$

D. $-4xy-7y^2+3x^2$

Part A

__50__ marks

TEACHER'S COPY

Value

PART B: WRITTEN RESPONSE QUESTIONS

Partial marks will be assigned for any correct work shown even if the final answer is incorrect.

(2) 1. If 35% of a number is 175, what is the original number?

$$\frac{35}{100}=\frac{175}{x}$$
$$35x=17\,500$$
$$\frac{35x}{35}=\frac{17\,500}{35}$$
$$x=500$$

The original number is 500.

(2) 2. Simplify $\frac{15}{16}\div\frac{5}{6}+\frac{9}{10}\times 30-\frac{4}{5}$.

$$\frac{15}{16}+\frac{5}{6}+\frac{9}{10}\times 30-\frac{4}{5}=\frac{\cancel{15}^{3}}{\cancel{16}_{8}}\times\frac{\cancel{6}^{3}}{\cancel{5}_{1}}+\frac{9}{\cancel{10}_{1}}\times\cancel{30}^{3}-\frac{4}{5}$$
$$=\frac{9}{8}+27-\frac{4}{5}$$
$$=1\frac{1}{8}+27-\frac{4}{5}$$
$$=28\frac{1}{8}-\frac{4}{5}$$
$$=27\frac{9}{8}-\frac{4}{5}$$
$$=27\frac{45}{40}-\frac{32}{40}$$
$$=27\frac{13}{40}$$

3. Complete the following.

(1) a. Express $\frac{3}{16}$ as a percent.

$$\frac{3}{16} = 0.1875$$

$$= 18.75\% \text{ or } 18\frac{3}{4}\%$$

$$\begin{array}{r} 0.1875 \\ 16\overline{)3.0000} \\ 16 \\ \hline 140 \\ 128 \\ \hline 120 \\ 112 \\ \hline 80 \\ 80 \\ \hline 0 \end{array}$$

(1) b. Express $\frac{5}{8}$ as a decimal number.

$$\frac{5}{8} = 0.625$$

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ 48 \\ \hline 20 \\ 16 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$$

(1) c. Express 0.125 as a basic fraction.

$$0.125 = \frac{125}{1000}$$

$$= \frac{1}{8}$$

(1) d. Express 0.256 as a percent.

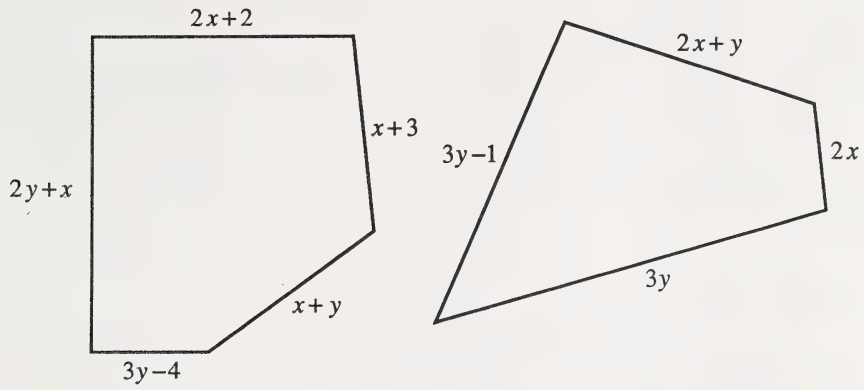
$$0.256 = 0.256 \times 100$$

$$= 25.6\%$$

TEACHER'S COPY

④ 4. Subtract the perimeter of Polygon II from the perimeter of Polygon I.

Polygon I **Polygon II**



Perimeter of Polygon I

$$2x+2+x+3+x+y+3y-4+2y+x = 2x+x+x+x+y+3y+2y+2+3-4$$
$$= (5x+6y+1) \text{ units}$$

Perimeter of Polygon II

$$2x+y+2x+3y+3y-1 = 2x+2x+y+3y+3y-1$$
$$= (4x+7y-1) \text{ units}$$

Difference of Polygon I and Polygon II

$$(5x+6y+1)-(4x+7y-1) = 5x+6y+1-4x-7y+1$$
$$= 5x-4x+6y-7y+1+1$$
$$= (x-y+2) \text{ units}$$

Value

5. Give a complete factored form for each of the following.

(2) a. $16a^2+48ab-16a^2b^2 = 16a\left(a+3b-ab^2\right)$

(2) b. $x^2-8x+16 = (x-4)(x-4)$
$= (x-4)^2$

(2) c. $5y^2+5y-30 = 5\left(y^2+y-6\right)$
$= 5(y+3)(y-2)$

(2) d. $x^2-13x+30 = (x-10)(x-3)$

(2) e. $9x^2-12x+4 = (3x-2)(3x-2)$

(2) f. $16y^4-25z^2 = \left(4y^2+5z\right)\left(4y^2-5z\right)$

6. Simplify.

(1) a. $\dfrac{-48a^7}{3a^5} = \dfrac{-48}{3}\times\dfrac{a^7}{a^5}$
$= -16a^2$

(1) b. $10x-4c+9-7x-3c-12 = 10x-7x-4c-3c+9-12$
$= 3x-7c-3$

(2) c. $-4b\left(-3bc^2+by-3\right) = 12b^2c^2-4b^2y+12b$

(2) d. $\left(-x^2-4c+5\right)-\left(-3x^2+4c+10\right) = -x^2-4c+5+3x^2-4c-10$
$= -x^2+3x^2-4c-4c+5-10$
$= 2x^2-8c-5$

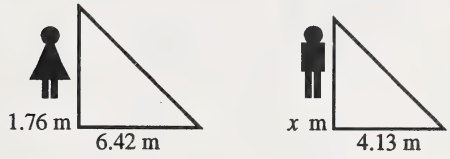
TEACHER'S COPY

(3) 7. Match each term with the correct definition. Place the correct letter in the blank provided.

a. transversal
b. adjacent angles
c. perpendicular lines
d. similar triangles
e. supplementary angles
f. diagonal

___e___ two angles which add to 180°
_____ triangles that have corresponding angles congruent
_____ triangle that is also an equilateral triangle
___a___ line which intersects two or more parallel lines
_____ triangle that has no sides that are equal
___f___ line that joins two nonconsecutive vertices
___b___ angles that have the same vertex and a common arm
___c___ form at least one right angle
___d___ triangles that have corresponding angles and corresponding sides congruent
_____ angle that measures more than 90°

(3) 8. June, who is 1.76 m tall, has a shadow of 6.42 m. At the same time of day, Jack has a shadow of 4.73 m. How tall is Jack? (Round your answer to one decimal place.)



$$\frac{1.76}{6.42} = \frac{x}{4.73}$$

$$6.42x = 1.76(4.73)$$

$$6.42x = 8.3248$$

$$\frac{6.42x}{6.42} = \frac{8.3248}{6.42}$$

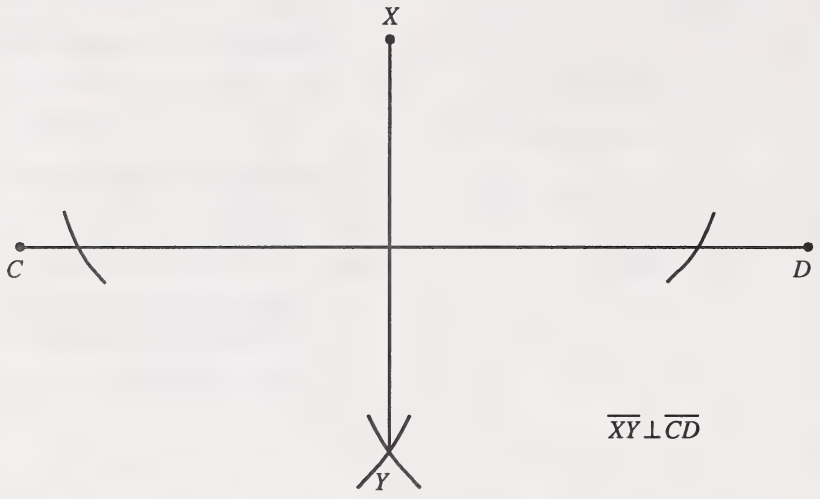
$$x \doteq 1.296\,697\,819$$

$$\doteq 1.3$$

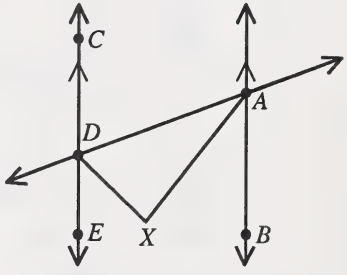
Jack is approximately 1.3 m tall.

TEACHER'S COPY

(3) 9. Draw $\overline{CD}$ which is 12 cm long. Choose a point X not on $\overline{CD}$ and above $\overline{CD}$. Using a compass and a straight edge only, construct a perpendicular to $\overline{CD}$ from point X. Show all the construction arcs.



(3) 10. Use the diagram to find the measure of $\angle XAB$, where $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$, $\overline{DX}$ bisects $\angle ADE$, $\angle ADE = 114°$, and $\angle AXD = 83°$.



$\angle ADX = \frac{1}{2} \times 114°$

$= 57°$

$\angle ADX + \angle DXA + \angle DAX = 180°$

$57° + 83° + \angle DAX = 180°$

$140° + \angle DAX = 180°$

$\angle DAX = 180° - 140°$

$\angle DAX = 40°$

$\angle DAX = 180° - 114°$

$= 66°$ (same side interior angles are supplementary)

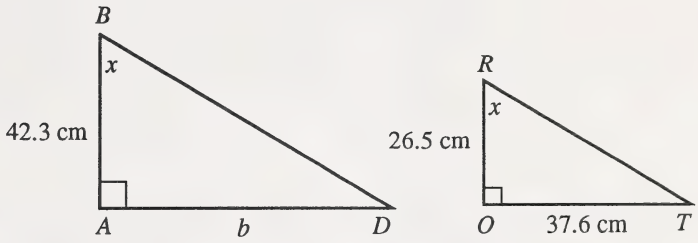
$\angle XAB = 66° - 40°$

$= 26°$

TEACHER'S COPY

(3) 11. ΔBAD is similar to ΔROT. $\angle A = \angle O = 90°$ and $\angle B = \angle R$. If $AB = 42.3$ cm and $RO = 26.5$ cm, find the measure of $\overline{AD}$ when $OT = 37.6$ cm. Round your answer to one decimal place.

$$\frac{b}{37.6} = \frac{42.3}{26.5}$$
$$26.5b = 37.6 \times 42.3$$
$$26.5b = 1590.48$$
$$\frac{26.5b}{26.5} = \frac{1590.48}{26.5}$$
$$b \doteq 60.018113\,21$$
$$\doteq 60.0$$



The measure of line segment AD is about 60.0 cm.

(2) 12. The measure of an angle plus three times its complement is 144°. Determine the measure of the angle.

Let the angle be x. The complement of x is $(90 - x)°$.

Therefore, $x + 3(90 - x) = 144$
$$x + 270 - 3x = 144$$
$$-2x + 270 = 144$$
$$-2x = 144 - 270$$
$$-2x = -126$$
$$\frac{-2x}{-2} = \frac{-126}{-2}$$
$$x = 63$$

The required angle measures 63° and its complement is 27°.

(3) 13. Rewrite $-20-4y=8x$ in $y=mx+b$ form; then find the slope and the y-intercept for this equation.

$$-20-4y=8x$$
$$-4y=8x+20$$
$$\frac{-4y}{-4}=\frac{8x}{-4}+\frac{20}{-4}$$
$$y=-2x-5$$

The slope is -2.
The y-intercept is -5.

(2) 14. a. If a graph passes through $A(4,9)$ and $B(-3,-5)$, what is the slope of the line?

$$\text{slope or } m=\frac{y_2-y_1}{x_2-x_1}$$
$$=\frac{-5-9}{-3-4}$$
$$=\frac{-14}{-7}$$
$$=2$$

The slope is 2.

(1) b. What is the slope of the line that is perpendicular to the line passing through $A(4,9)$ and $B(-3,-5)$?

$$\text{slope or } m=\frac{y_2-y_1}{x_2-x_1}$$
$$=\frac{-5-9}{-3-4}$$
$$=\frac{-14}{-7}$$
$$=2$$

The slopes of perpendicular lines are negative reciprocals. The negative reciprocal of 2 is $-\frac{1}{2}$. The slope of the perpendicular line is $-\frac{1}{2}$.

TEACHER'S COPY

(3) 15. Write the equation $\frac{-4}{3}y=\frac{-5}{2}x-\frac{3}{5}$ in the form $ax+by+c=0$.

$$\frac{-4y}{3}=\frac{-5x}{2}-\frac{3}{5}$$

$$30\left(\frac{-4y}{3}\right)=30\left(\frac{-5x}{2}\right)-30\left(\frac{3}{5}\right)$$

$$-40y=-75x-18$$

$$75x-40y+18=0$$

(4) 16. A section of flooring to be carpeted is in the shape of a right triangle. One side of the floor is 5.2 m long and the hypotenuse is 11.3 m long. Find the length of the third side. (Round your answer to the nearest thousandth.)

$$a^2+b^2=c^2$$

$$(5.2)^2+b^2=(11.3)^2$$

$$27.04+b^2=127.69$$

$$b^2=127.69-27.04$$

$$b^2=100.65$$

$$\sqrt{b^2}=\sqrt{100.65}$$

$$b\doteq 10.032\,447\,36$$

$$\doteq 10.032$$

The length of the third side is about 10.032 m.

(3) 17. Find the values for f, g, and h in the chart.

Cost	125	f	500	625	875	1000	h	1875
Number	1	2	4	5	g	8	10	15

$f = 2 \times 125$
$= 250$

$g = 875 \div 125$
$= 7$

$h = 10 \times 125$
$= 1250$

(3) 18. If c varies inversely as d varies and $c = 25$ when $d = 17$, find d when $c = 14$. (Round the answer to the nearest whole number.)

$$\frac{c_1}{c_2} = \frac{d_1}{d_2}$$
$$c_1 d_1 = c_2 d_2$$
$$25 \times 17 = 14 \times d_2$$
$$14 d_2 = 425$$
$$\frac{14 d_2}{14} = \frac{425}{14}$$
$$d_2 \doteq 30.357\,142\,86$$
$$\doteq 30$$

or

$$cd = k$$
$$k = 25 \times 17$$
$$k = 425$$
$$cd = k$$
$$14d = 425$$
$$\frac{14d}{14} = \frac{425}{14}$$
$$d \doteq 30.357\,142\,86$$
$$\doteq 30$$

The value for d is approximately 30 when $c = 14$.

TEACHER'S COPY

19. A candy company wants to check the consistency of the weights of the chocolate bars it makes. One particular day sixteen bars were chosen randomly and weighed. The weights of the bars in grams are listed as follows.

52	62	59	57	54	53	56	55
58	61	60	58	54	54	49	53

(3) a. Organize this data into a stem-and-leaf plot.

49	52	53	53	54	54	54	55
56	57	58	58	59	60	61	62

4	9
5	2 3 3 4 4 4 5 5 6 7 8 8 9
6	0 1 2

(3) b. Find the mean, median, and mode for this data. If necessary round the answer to one decimal place.

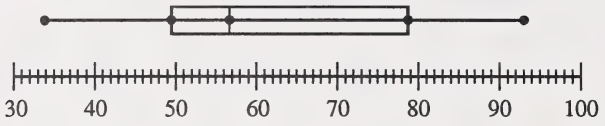
$$\text{Mean} = \frac{\text{sum of all elements}}{\text{number of elements}}$$
$$= \frac{895}{16}$$
$$= 57.8125$$
$$\doteq 57.8 \text{ g}$$

$$\text{Median} = \frac{55+56}{2}$$
$$= \frac{111}{2}$$
$$= 55.5 \text{ g}$$

Mode = 54 g

TEACHER'S COPY

(3) 20. Answer the following questions for the box-and-whisker plot.



a. What is the median?

The median is 57.

b. What is the lower quartile?

The lower quartile is 50.

c. What is the upper quartile?

The upper quartile is 29.

Part B

__75__ marks

TEACHER'S COPY

MATHEMATICS 13

FINAL TEST

GENERAL INSTRUCTIONS

1. The time limit for this test is **two hours**.

2. This test consists of two parts. Complete both parts.

3. Write all answers legibly in pen or pencil.

4. Calculators that are not programmed may be used.

5. For written response questions, be sure to show all your work. Part marks will be given for partially correct solutions.

6. It is better to complete all the work you know well first. After that you can deal with the difficult ones. Do not spend too much time on any one problem. If you finish the test before the two-hour time limit, go back and check your answers.

TOTAL MARKS: 125

PART A: Multiple Choice – 50 marks

PART B: Written Response – 75 marks

Value

50

PART A: MULTIPLE CHOICE

In each of the questions, indicate the correct answer by placing the appropriate letter A, B, C, or D in the blank provided.

Example:

_B_ The value of 2^3 is

A. 6

B. 8

C. -8

D. -6

_____ 1. In $-8, -6, 2, 10, 10, 14, 34$, the number 10 is referred to as the

A. range

B. average

C. mean

D. mode

_____ 2. The range for $-5, -3, 0, 4, 9, 10$, and 13 is

A. 18

B. 8

C. 13

D. 5

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

_____ 3. Two straight lines are parallel. If the slope of one of these lines is -5, the slope of the other line is

A. 5

B. $\frac{1}{5}$

C. -5

D. $-\frac{1}{5}$

_____ 4. What is the degree of the given polynomial?
$-6a^3b^2c+4ab^2c-3$

A. 6

B. 10

C. 3

D. 4

_____ 5. The mean for $-3, -1, 0, 5, 6,$ and 8 is

A. 0

B. 2.5

C. -4

D. -2.5

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

_____ 6. The value for x in the table is

4	6	8	x	15
16	24	32	44	60

A. 10

B. 11

C. 12

D. 14

_____ 7. According to the number of terms in $\frac{-2ab}{5}$, what is the expression called?

A. quotient

B. binomial

C. monomial

D. trinomial

_____ 8. An isosceles triangle always has the following.

A. all angles are less than 90°

B. two angles are less than 90° and one angle is greater than 90°

C. all three sides have different lengths

D. at least two sides have equal lengths

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

_____ 9. Two straight lines are perpendicular. If the slope of one of these lines is $-\frac{5}{4}$, the slope of the other line is

A. $\frac{4}{5}$

B. -1

C. $-\frac{4}{5}$

D. $\frac{5}{4}$

_____ 10. The value for y in the table is

-5	-1	0	1	5
10	2	0	-2	y

A. -5

B. 5

C. -10

D. 10

_____ 11. Which of the following is the square of the hypoteneuse of a right triangle equal to?

A. the square root of the base

B. the sum of the squares of the other two sides

C. the product of the other two sides

D. the square root of the altitude

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

_____ 12. The simplified form for $(-6a+4b-10c)+(7a-5b+3c)-(-a-5b-4c)$ is

A. $-14a+7b-17c$

B. $4a+2b-3c$

C. $2a+4b-3c$

D. $3a+3c-3b$

_____ 13. In the diagram, which property makes these two triangles congruent?

A. SSS

B. ASA

C. SAS

D. AAS

_____ 14. In 14, 18, 18, and 35, 18 can be referred to as the median or the

A. mean

B. bimodal

C. range

D. mode

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

_____ 15. In $-4z$, what does z represent?

A. term

B. literal coefficient

C. sum

D. numerical coefficient

_____ 16. A straight line is perpendicular to the x-axis. What can be said about this slope?

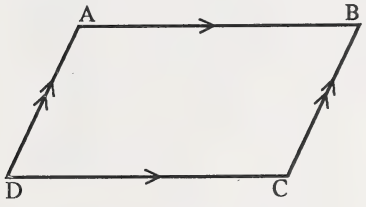
A. The slope is undefined.

B. The slope is positive.

C. The slope is zero.

D. The slope is negative.

_____ 17. Quadrilateral $ABCD$ is a parallelogram.



$m\angle B + m\angle C = 180°$ because

A. two angles are corresponding angles

B. two angles are complimetary

C. two angles are vertically opposite angles

D. two angles are same-side interior angles

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

_____ 18. Which of the following is an integer?

A. $3 \div 0$

B. $-16 \div (-8)$

C. $1 \div (-3)$

D. $4 \div 15$

_____ 19. The $y = mx + b$ form for $-20 + 5y = 10x$ is

A. $y = 2x + 4$

B. $5y = 10x + 20$

C. $y = 10x - 20$

D. $5y = -20 - 10x$

_____ 20. Find the value for $\frac{-2a^2bc^2}{30a}$ given $a = -3$, $b = 3$, and $c = -5$.

A. 1350

B. -15

C. 15

D. 5310

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

_____ 21. If y varies directly as x and $y = 36$ when $x = 6$, the constant of variation is

A. 6

B. 8

C. 12

D. 2

_____ 22. In simplest form, $\frac{2^{-3}}{2^{-5}}$ is

A. $\frac{1}{4}$

B. 4

C. $\frac{1}{2}$

D. 2

_____ 23. The product of $(b-7)(b+4)$ is

A. $b^2+3b+28$

B. $b^2+3b-28$

C. $b^2-3b-28$

D. $b^2+11b+28$

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

_____ 24. The numerical coefficient in $-a^4b^2c$ is

A. 1

B. 4

C. 2

D. -1

_____ 25. The evaluated form for $(-3)^3 \times (-3)^0$ is

A. 9

B. -27

C. 27

D. 0

_____ 26. Determine an equation relating a to b if a is directly proportional to b, when $b = 3.2$ and $a = 22.4$.

A. $a = 7b$

B. $b = 7a$

C. $7 = ab$

D. $ab = 7$

Name of Student _____ Student I.D. # _____

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_____ 27. How many variables are there in $4a^3c^3x^2+2ac-3x+14abx$?

A. 9

B. 2

C. 4

D. 3

_____ 28. The quotient for $\frac{16x^3y^6z^4+10x^3y^3-2x^2y^3}{2x^2y^3}$ is

A. $8xyz^4+5x^2$

B. $16x^2y+10x-1$

C. $8xyz+5xy-2xy^2$

D. $8xy^3z^4+5x-1$

_____ 29. When a line is parallel to the y-axis on a coordinate plane, the slope is

A. undefined

B. negative

C. zero

D. positive

Name of Student _____ Student I.D. # _____

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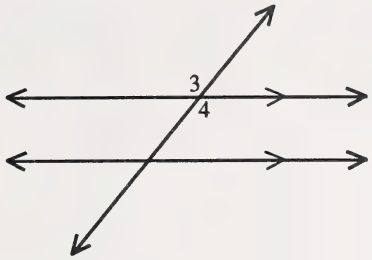
_____ 30. The special name given to $\angle 3$ and $\angle 4$ in the diagram is



A. corresponding

B. vertically opposite

C. supplementary

D. alternate

_____ 31. The factored form for $12y^2 - 3y$ is

A. $2y^2(6y-1)$

B. $3y(4y-1)$

C. $2y(6y-1)$

D. $4y(2y-1)$

_____ 32. The slope of a straight-line graph that falls from left to right is

A. unequal

B. positive

C. equal

D. negative

Name of Student _____ Student I.D. # _____

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_____ 33. Which of the following equations show partial variation?

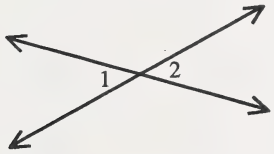
A. $C = m + bx$

B. $C = \frac{x}{y}$

C. $xy = C$

D. $\frac{C}{x} = \frac{1}{y}$

_____ 34. In the diagram the angles labelled $\angle 1$ and $\angle 2$ are called



A. adjacent angles

B. alternate angles

C. vertically opposite angles

D. corresponding angles

_____ 35. Which of the following is undefined?

A. $\frac{45}{x}$ where $x = -1$

B. $\frac{45}{x}$ where $x = 0$

C. $\frac{45}{x}$ where $x \neq 0$

D. $\frac{45}{x}$ where $x = 1$

_____ 36. In a survey, all possible samples belong to the

A. probability

B. random number table

C. data

D. population

Name of Student _____ Student I.D. # _____

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_____ 37. The lowest common denominator for $\frac{4}{5}$, $\frac{3}{10x}$, and $\frac{1}{2x^2}$ is

A. $10x^2$

B. $50x^2$

C. $20x^3$

D. $100x$

_____ 38. In $\overrightarrow{AB}$, the A is called a (an)

A. start

B. line

C. endpoint

D. ray

_____ 39. Which of the following gives a full word description for $\overline{AB} \perp \overleftrightarrow{CD}$?

A. Ray AB is parallel to line CD.

B. Line segment AB is perpendicular to line CD.

C. Line segment AB is parallel to line CD.

D. Line segment AB is not parallel to line CD.

_____ 40. The point $F(-4, -1)$ is in

A. Quadrant III

B. Quadrant II

C. Quadrant IV

D. Quadrant I

_____ 41. If the probability of an event is 1, this is also known as

A. an impossible event

B. a certain event

C. a statistics event

D. a biased event

Name of Student _____ Student I.D. # _____

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_____ 42. When graphing a relation, what does the vertical axis represent?

A. the independent variable

B. the related variable

C. the dependent variable

D. the unrelated variable

_____ 43. To select a red ball from a bag of white balls is a (an)

A. impossible event

B. possible event

C. uncertain event

D. certain event

_____ 44. If all four arithmetical operations are found in a number expression, the one which must be done first is

A. multiplication

B. addition

C. division

D. subtraction

_____ 45. The two areas on a coordinate plane intersect at the

A. origin

B. ordered pair

C. quadrant

D. dependent variable

Name of Student _____ Student I.D. # _____

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_____ 46. Simplify this expression.

$$\frac{4x^2y^3}{15z^4} \div \frac{12xy^2}{5z^2}$$

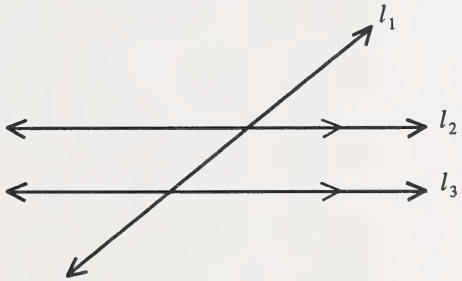
A. $\frac{4x}{5y}$

B. $\frac{xy}{9z^2}$

C. $\frac{3z}{20x^2y}$

D. $\frac{48z^3}{75xy}$

_____ 47. What is the name of l_1 in this diagram?



A. intersector

B. bisector

C. transversal

D. perpendicular

_____ 48. What is the minimum number of points needed to make a straight-line graph?

A. one

B. three

C. four

D. two

Name of Student _____ Student I.D. # _____

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_____ 49. Monomials, binomials, and trinomials can be referred to generally as

A. polynomials

B. four-term polynomials

C. terms

D. factors

_____ 50. Rearrange $-4xy+3x^2-7y^2$ in descending order of powers.

A. $3x^2-4xy-7y^2$

B. $-7y^2+3x^2-4xy$

C. $-7y^2-4xy+3x^2$

D. $-4xy-7y^2+3x^2$

Part A

_____ marks

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

PART B: WRITTEN RESPONSE QUESTIONS

Partial marks will be assigned for any correct work shown even if the final answer is incorrect.

(2) 1. If 35% of a number is 175, what is the original number?

(2) 2. Simplify $\frac{15}{16} \div \frac{5}{6} + \frac{9}{10} \times 30 - \frac{4}{5}$.

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

3. Complete the following.

(1) a. Express $\frac{3}{16}$ as a percent.

(1) b. Express $\frac{5}{8}$ as a decimal number.

(1) c. Express 0.125 as a basic fraction.

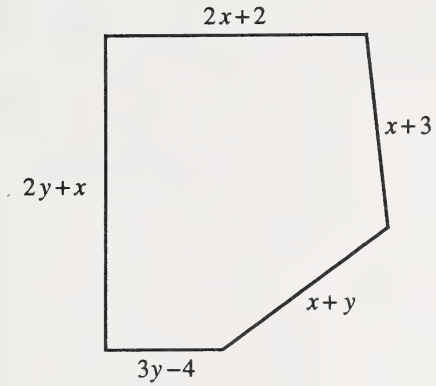
(1) d. Express 0.256 as a percent.

Name of Student _____ Student I.D. # _____

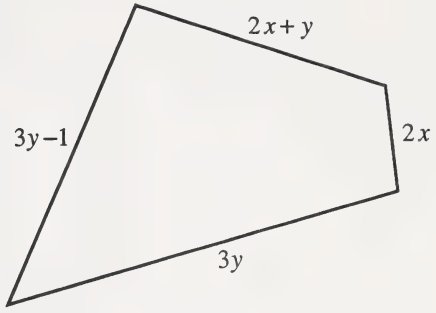
Name of School _____ Date _____

(4) 4. Subtract the perimeter of Polygon II from the perimeter of Polygon I.

Polygon I



Polygon II



Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

5. Give a complete factored form for each of the following.

(2) a. $16a^2+48ab-16a^2b^2$

(2) b. $x^2-8x+16$

(2) c. $5y^2+5y-30$

(2) d. $x^2-13x+30$

(2) e. $9x^2-12x+4$

(2) f. $16y^4-25z^2$

6. Simplify.

(1) a. $\dfrac{-48a^7}{3a^5}$

(1) b. $10x-4c+9-7x-3c-12$

(2) c. $-4b\left(-3bc^2+by-3\right)$

(2) d. $\left(-x^2-4c+5\right)-\left(-3x^2+4c+10\right)$

Name of Student _____ Student I.D. # _____

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(3) 7. Match each term with the correct definition. Place the correct letter in the blank provided.

a. transversal
b. adjacent angles
c. perpendicular lines
d. similar triangles
e. supplementary angles
f. diagonal

_____ two angles which add to 180°
_____ triangles that have corresponding angles congruent
_____ triangle that is also an equilateral triangle
_____ line which intersects two or more parallel lines
_____ triangle that has no sides that are equal
_____ line that joins two nonconsecutive vertices
_____ angles that have the same vertex and a common arm
_____ form at least one right angle
_____ triangles that have corresponding angles and corresponding sides congruent
_____ angle that measures more than 90°

(3) 8. June, who is 1.76 m tall, has a shadow of 6.42 m. At the same time of day, Jack has a shadow of 4.73 m. How tall is Jack? (Round your answer to one decimal place.)

Name of Student _____ Student I.D. # _____

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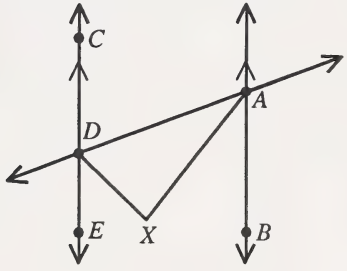
(3) 9. Draw $\overline{CD}$ which is 12 cm long. Choose a point X not on $\overline{CD}$ and above $\overline{CD}$. Using a compass and a straight edge only, construct a perpendicular to $\overline{CD}$ from point X. Show all the construction arcs.

(3) 10. Use the diagram to find the measure of $\angle XAB$, where $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$, $\overline{DX}$ bisects $\angle ADE$, $\angle ADE = 114°$, and $\angle AXD = 83°$.



Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

(3) 11. ΔBAD is similar to ΔROT. $\angle A = \angle O = 90°$ and $\angle B = \angle R$. If $AB = 42.3$ cm and $RO = 26.5$ cm, find the measure of $\overline{AD}$ when $OT = 37.6$ cm. Round your answer to one decimal place.

(2) 12. The measure of an angle plus three times its complement is 144°. Determine the measure of the angle.

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

(3) 13. Rewrite $-20-4y=8x$ in $y=mx+b$ form; then find the slope and the y-intercept for this equation.

(2) 14. a. If a graph passes through $A(4,9)$ and $B(-3,-5)$, what is the slope of the line?

(1) b. What is the slope of the line that is perpendicular to the line passing through $A(4,9)$ and $B(-3,-5)$?

Name of Student _____ Student I.D. # _____

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(3) 15. Write the equation $\frac{-4}{3}y = \frac{-5}{2}x - \frac{3}{5}$ in the form $ax + by + c = 0$.

(4) 16. A section of flooring to be carpeted is in the shape of a right triangle. One side of the floor is 5.2 m long and the hypotenuse is 11.3 m long. Find the length of the third side. (Round your answer to the nearest thousandth.)

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

(3) 17. Find the values for f, g, and h in the chart.

Cost	125	f	500	625	875	1000	h	1875
Number	1	2	4	5	g	8	10	15

$f =$

$g =$

$h =$

(3) 18. If c varies inversely as d varies and $c = 25$ when $d = 17$, find d when $c = 14$. (Round the answer to the nearest whole number.)

Name of Student _____ Student I.D. # _____

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19. A candy company wants to check the consistency of the weights of the chocolate bars it makes. One particular day sixteen bars were chosen randomly and weighed. The weights of the bars in grams are listed as follows.

52	62	59	57	54	53	56	55
58	61	60	58	54	54	49	53

(3) a. Organize this data into a stem-and-leaf plot.

(3) b. Find the mean, median, and mode for this data. If necessary round the answer to one decimal place.

Name of Student _____ Student I.D. # _____

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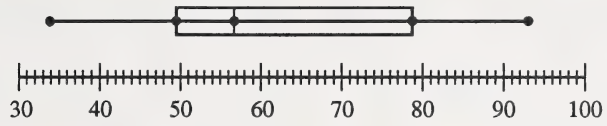
(3) 20. Answer the following questions for the box-and-whisker plot.



a. What is the median?

b. What is the lower quartile?

c. What is the upper quartile?

Part B

_____ marks

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

This is a course designed in a new distance-learning format, so we are interested in your responses. Your constructive comments will be greatly appreciated so that a future revision may incorporate any necessary improvements.

TEACHER QUESTIONNAIRE FOR MATHEMATICS 13

Teacher's Name _____ Area of Expertise _____

School Name _____ Date _____

Design

1. The units follow a definite systematic design. Did you find it easy to follow?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

2. Did your observations reveal that the students found the design easy to follow?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

3. Did you find the Learning Facilitator's Manual helpful?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

4. Part of the design involves stating the objectives in student terms. Do you feel this helped the students understand what they were going to learn?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

5. The topics include self-assessment questions. These questions are to help clarify and reinforce the instructional materials. Did students use the answers effectively?

❑ Yes ❑ No If no, explain.

_____

_____

6. Did the Extra Help and Extensions prove to be helpful?

❑ Yes ❑ No If no, explain.

_____

_____

7. Were students motivated to try the Extra Help and Extensions?

❑ Yes ❑ No If no, give details.

_____

_____

8. Suggestions for computer, video, and audio activities are included in the course. Were your students able to use these activities?

❑ Yes ❑ No Comment on the lines below.

_____

_____

9. Were the assignments appropriate?

❑ Yes ❑ No If no, give details.

_____

_____

10. Did you fax assignments?

❑ Yes ❑ No

11. If you did fax, did you get satisfactory results from using this procedure?

❑ Yes ❑ No If no, give details.

_____

_____

Instruction

1. Did you find the instruction clear?

❑ Yes ❑ No If no, give details.

_____

_____

2. Did your observations reveal that the students found the instruction interesting?

❑ Yes ❑ No If no, give details.

_____

_____

3. Did you find the instruction adequate?

❑ Yes ❑ No If no, give details.

_____

_____

4. Was the reading level appropriate?

❑ Yes ❑ No If no, give details.

_____

_____

5. Was the workload adequate?

❑ Yes ❑ No If no, give details.

_____

_____

6. Was the content accurate and current?

☐ Yes ☐ No If no, give details.

_____

_____

7. Did the content flow consistently and logically?

☐ Yes ☐ No If no, give details.

_____

_____

8 Was the transition between booklets smooth?

☐ Yes ☐ No If no, give details.

_____

_____

9. Was the transition between print and media smooth?

☐ Yes ☐ No If no, give details.

_____

_____

Additional Comments

_____

_____

_____

_____

_____

_____

When you have completed this questionnaire, please mail it to the following address:

Design Department
Alberta Distance Learning Centre
Box 4000
Barrhead, Alberta
T0G 2P0

L.R.D.C.
Producer

Mathematics 13

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THIRD EDITION
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